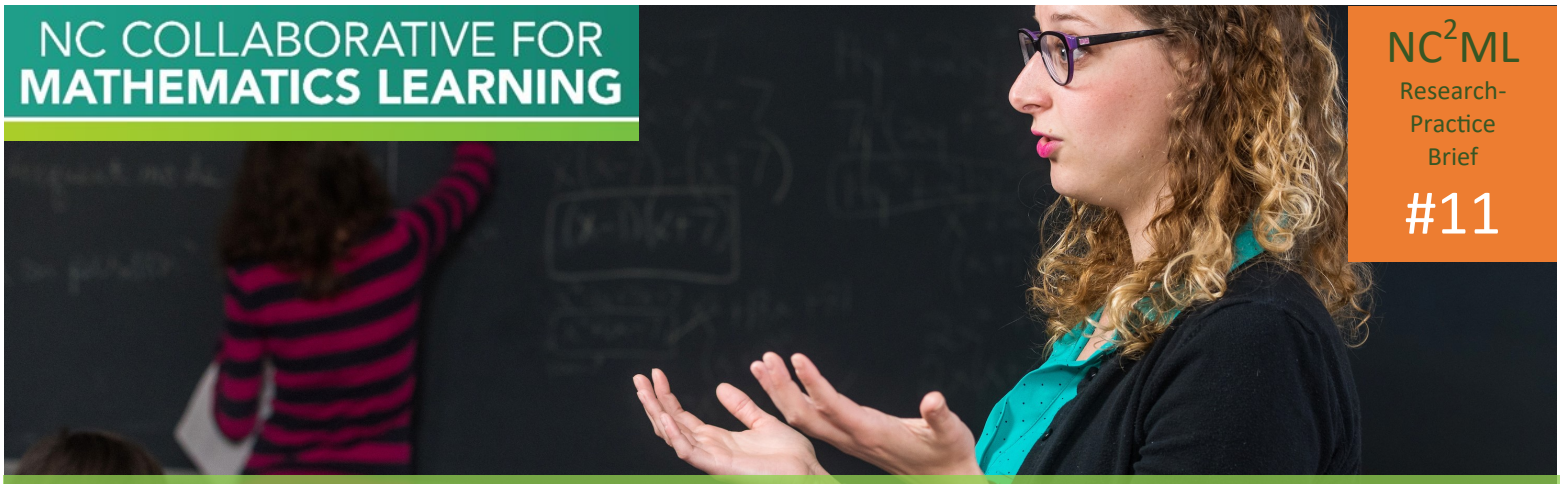
NC COLLABORATIVE FOR MATHEMATICS LEARNING



NC²ML Research-Practice Brief #11

NC Math 2 - Trigonometry

NC MATH 2 – UNIT 5, TRIGONOMETRY

In NC Math 2 trigonometric functions, **sine**, **cosine**, and **tangent**, are introduced for the first time and is done through the study of ratios of side lengths of right triangles. As such, students will verify that ratios of side lengths of similar triangles are proportional as they equal the trigonometric function value of the same angle measure (NC.M2.G-SRT.6). These definitions will be applied to contextual problems and proofs about triangles (NC.M2.G-SRT.4 & 8, NC.M2.G-CO.10).

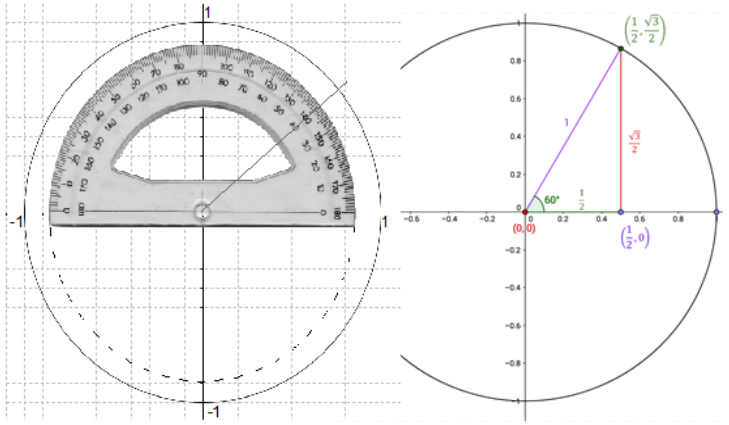
In NC Math 3 students will expand their understanding of ***sine*** and ***cosine*** as functions with domains that can be related to the set of all possible angle measures, while working with the units of radians and connecting to the representation of the unit circle (NC.M3.F-TF). Until then, **NC Math 2 students will only focus on trigonometric values of acute angles within right triangles.**

TRIGONOMETRIC RATIOS AS OPERATIONS

One of the reasons that trigonometric functions are difficult for many students is that it is the first time they see an operation in which the actions to take may not be clear (Weber et al., 2008). For example, when one sees a square root or a cubic expression, students know what actions to take to simplify the statement. Trigonometric operations can be thought of as operations on angles. For example, sine can be thought of as the process of using a protractor to draw a ray emanating from the origin of the unit circle, locating the point of intersection of the ray and the unit circle and determining the y-value of that intersection (Weber et al., 2008). *See the "Computing Sine and Cosine" figure.*

Just because students do not carefully study the unit circle until NC Math 3 does not mean such connections should be avoided in Math 2. The understanding of these operations (***sine*** and ***cosine***) as functions are the focus of NC Math 3, but NC Math 2 students will *also* need to understand that when calculating the ***sine***, ***cosine,*** or ***tangent*** value of an angle, that output is *predictable*. Every angle will have exactly one trigonometric value.

COMPUTING SINE AND COSINE



SIMILARITY AND TRIGONOMETRY

The study of trigonometric ratios extends students' work in measurement, multiplication, and division from the elementary grades; ratios and proportional relationships in middle grades; and linear rates of change in NC Math 1. From these previous experiences students understand that a ratio expresses the comparison between two quantities. In the case of trigonometric ratios, we are comparing the relationship between two sides of any right triangle. Using their understanding of similar triangles, students can discover that at any acute vertex angle of a triangle, no matter the size of the legs, these ratios of the sides are always equal. Developing this idea of a unit rate using trigonometric ratios stems naturally from students' exploration of linear rates of change and exponential rates of growth from NC Math 1.

In NC Math 3 students will need to be able to connect back to

the trigonometric ratios they learned in Math 2. While there is little research specifically addressing right triangle trigonometry, an abundance of research points to the ineffectiveness of memorization techniques in mathematics (i.e. Boaler, 1998). Students need to develop a deep conceptual understanding of mathematical ideas like

NC.M2.G-SRT.6

Verify ***experimentally*** that the side ratios in similar right triangles are properties of the angle measures in the triangle, due to the preservation of angle measure in similarity. ***Use this discovery to develop definitions*** of the trigonometric ratios for acute angles.

trigonometric ratios in order to continue expanding their knowledge of related concepts. Therefore it is imperative that students *discover* these ratios instead of only learning a mnemonic to help them memorize them. Online tools, like Geogebra, are a great way to build triangles that you can manipulate to investigate the sine, cosine, and tangent ratios. In fact, look for activities that others have already created like Discovering Trig Ratios worksheet. Giving students time to understand what they are actually finding when they type $\sin 15°$ into the calculator will help them in NC Math 3 when they transition into using the unit circle to evaluate values of trigonometric functions.

TRIGONOMETRY AND EQUIVALENT EXPRESSIONS

Once students have defined the ***sine*** and ***cosine*** ratios through investigating similarity in triangles, the ratio for the ***tangent*** of an angle can be derived.

$$tan(a) = \frac{\sin(a)}{\cos(a)} = \frac{\frac{opposite}{hypotenuse}}{\frac{adjacent}{hypotenuse}} = \frac{opposite}{hypotenuse} \cdot \frac{hypotenuse}{adjacent} = \frac{opposite}{adjacent}$$

This derivation highlights how mathematical definitions can connect using the substitution of an equivalent expressions. This use of equivalent expressions may seem obvious, but such uses of equivalence and equality need to be modeled while making the reasoning explicit for students. It is not uncommon for students to develop partial understandings of the equal sign, often thinking of it as a message to "compute", as such it is always important to draw attention to the use of equivalencies to support students in their continued development of this important mathematical relationship (Knuth et al., 2008)

In addition to making sense of trigonometric operations and relationships, NC Math 2 students will be expected to use them. Typically, this means solving problems or proving theorems by identifying a right triangle within the given context, then using a trigonometric ratio to find a missing value. Weber (2005) found that many students view trigonometric ratios as a "step-by-step prescription that are to be applied to an external cue" (p.103). Building the ratios through similar triangles, pointing out their "predictable" outputs, and exploring concepts of equality can help students build more well-rounded conceptions of trigonometry.

SPECIAL RIGHT TRIANGLES

Special triangles explored in NC Math 2 include equilateral triangles and right isoscelese triangles. It's important to note that adding an altitude to a non-right triangle, is the first step toward a deeper analysis of that triangle's properties. Students should see that given an equilateral triangle, identifying an altitude and using the AA~ property proves the two halves created by the altitude are similar triangles, with congruence then following from SAS (using the altitude as one of those sides). From here students can use the Pythagorean Theorem to justify and investigate the $1, \frac{1}{2}, \frac{\sqrt{3}}{2}$ ratios that exist within ***all*** equilateral triangle sides and altitudes.

Right isosceles triangles, whose side ratios of $1, 1, \sqrt{2}$ also follow from applying the equation of the Pythagorean Theorem. This very *geometric* investigation allows for students to be able to calculate the *trigonometric* values which will become the ***trigonometric benchmarks*** that students will be expected to recall and remember.

TRIGONOMETRIC BENCHMARKS

$$\sin(30°) = \frac{1}{2} \qquad \sin(45°) = \frac{\sqrt{2}}{2} \qquad \sin(60°) = \frac{\sqrt{3}}{2}$$

$$\cos(30°) = \frac{\sqrt{3}}{2} \qquad \cos(45°) = \frac{\sqrt{2}}{2} \qquad \cos(60°) = \frac{1}{2}$$

References

Boaler, J. (1998). Open and Closed Mathematics: Student Experiences and Understandings. *Journal for Research in Mathematics Education, 29*(1), 41-62.

Knuth, E. J., Alibali, M. W., Hattikudur, S., McNeil, N. M., & Stephens, A. C. (2008). The importance of equal sign understanding in the middle grades. Mathematics Teaching in the Middle School, 13 (9), 514–519.

Weber, K. (2005). Students' understanding of trigonometric functions. *Mathematics Education Research Journal*, *17*(3), 91-112.

Weber, K. Knott, L., & Evitts, T. (2008). Teaching trigonometric functions: Lessons learned from research. *Mathematics Teacher*, 102(2), 144-150.

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