NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC^{2}ML Research-Practice Brief #12

NC Math 2 – Probability

NC MATH 2 – UNIT 6 BEGINS IN 7TH GRADE

The big idea in the probability standards for NC Math 2 is for students to understand, explain, and use conditional probabilities. This includes understanding when and how to apply the addition and multiplications rules for probability.

There is a likelihood that NC Math 2 students have not worked with probability since 7th grade, where essentially all of the middle grades probability standards are clustered. Even so, students should be familiar with the definition for the **probability of an event** (i.e., *a measure of the likelihood that the event will occur)* as it is central to probability standards in 7th grade. The less familiar interpretation of the probability of an event is to see it *as a long-run relative frequency,* which has its foundation in 7th grade.

> ***NC.7.SP.6*** *Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.*

The cluster of 7.SP-standards involve creating probability models to run simulations of events and record the observed frequencies (**NC.7.SP.8.c**). These simulations are the "chance processes" of **NC.7.SP.6**, that produce the *experimental probabilities* calculated by the ratio of relative frequency to the total number of simulations. 7th grade work with proportions will help students be able to approximate the relative frequency given the (theoretical or observed) probability. See the North Carolina 7th Grade Standards for more details.

RANDOMNESS

This 7th grade cluster extends to **NC Math 2** as students again use probability models for simulations.

> ***NC.M2.S-IC.2*** *Use simulation to determine whether the experimental probability generated by sample data is consistent with the theoretical probability based on known information about the population.*

One distinction from the middle grades standards in this **NC Math 2** standard is the notion of *sample data* that is representative of a *population*. *Randomness* in sampling is vital to prevent bias, and truly represent the entire population. So what do we know about how students think about randomness? Research says that students sometimes have a *representativeness* view of randomness (Garfield, 1995). They expect all samples to closely "represent" the population. For example, consider the probability of getting heads when flipping a coin. Students may believe that landing on heads 80% of the time is equally likely for 10 flips as it is for 1000 flips. Also, with a representativeness view students' expect that the probability of getting a heads after landing on tails 4 times in a row is more likely than the outcome of tails. This is commonly referred to as Gambler's Fallacy (Jones & Thornton, 2005). Students are not wrong in realizing that 5 tails in a row are highly unlikely, however, they should also understand that coin flips are independent events. The conflict is distinguishing between the independence of 5 single coin flips versus the probability of the string of 5 flips. Konold et al. (1993) suggests engaging students in a conversation about the difference of these two situations. One way to do this is by posing the following "agree/disagree" problem:

> *Sam flips a coin 4 times and gets T, T, T, T. He claims that the probability of getting a T on the next flip is 50/50. Barb disagrees, and says that the probability will be much less since landing on "T" five times in a row is very slim. Who do you agree with? Why?*
>
> Both students are correct, however they are answering different problems. Sam is answering the probability of a coin landing on tails. Barb is answering the probability on landing on five tails in a row.

Introducing this cognitive conflict creates opportunities for classroom discussion that further develop students' understanding of randomness and probability. The concept of randomness becomes even more important in NC Math 3 when students extend their understanding by learning about random sampling techniques in statistics.

TOOLS FOR UNDERSTANDING PROBABILITY

The NC Math 2 Mathematical Resources for Instruction discusses the importance of students physically conducting simulations using tactile tools such as spinners, dice, or colored tiles in a bag. However, using technology allows students to quickly run large scale simulations and can be helpful in fostering discussions comparing experimental and theoretical probability thus allowing student to make sense of the connections between the two (Stohl, 2005). Consider a basketball player with a 60% shooting percentage. Using a weighted spinner (60% make/40% miss), students can simulate the number of shots made over the next 10 free throws, then compare their experimental results with the theoretical. Many students take these tools (like the spinner) at face value, not realizing they can be used to represent different situations. Another example of using a tool to represent something more meaningful is flipping a coin to represent the birth of a boy or girl. Check out the CPM Probability Tools if you don't have any of these hands-on tools for your classroom. There's also an adjustable spinner at NCTM's Illuminations website that is very user-friendly.

These simple tools can support students in understanding that **outcomes** of a chance process are the elements of the **samples space** of all possible outcomes, and that **events** are subsets of the sample space. Understanding these relationships as set and subset relationships is vital to meet **NC.M2.S-CP.1**, in which students describe events as results of the set operations of unions, intersections, and complements.

REPRESENTATIONS BUILD MEANING

Research has shown that tasks that involve "without replacement" can be more challenging for students because they often fail to attend to the changes in sample space (Tarr & Lannin, 2005). Actually simulating an experiment involving "without replacement" (e.g. marbles in a bag) allows students to understand how "not replacing" affects the probability of the next outcome.

Conditional probabilities can often be understood very naturally through contexts and the display of data within a two-way table. Experiences that connect with context and representations will be helpful in meeting standards **NC.M2.S-CP3-5** where the goals are for students to *understand, recognize, and explain the concepts* of **conditional probabilities** and **independence of events.** One classic example, is a two-way table of data on passengers who were on the Titanic, where survival is more likely for one set of passengers than another.

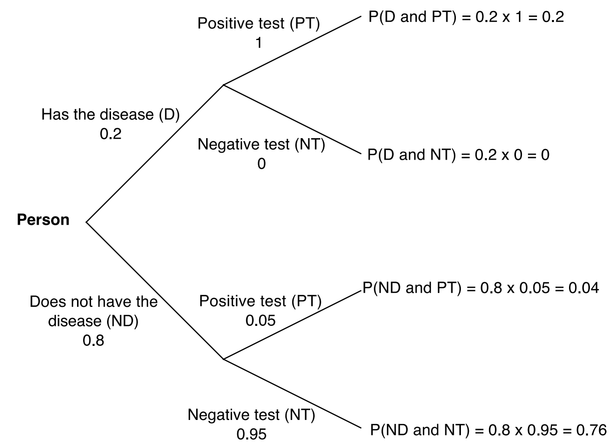
Passenger data from- www.icyousee.org/titanic.html	Men	Women	**Total**
Survived	128	304	**432**
Died	648	108	**756**
Total	**776**	**412**	**1188**

This representation highlights important relationships when thinking about conditional probabilities. For example, using the two-way table above, consider the following questions:

1. Calculate the probability for surviving the wreck.
2. What's the probability of survival given that you are a female passenger?
3. What's the probability of survival given that you are a male passenger?

Thinking about Representations of Conditional Probability
Why was the sample space different for each of the three questions above? What was helpful about the presentation of the data when answering these questions? What pieces of the 2-way table did you focus your attention? Why?

Another common and very helpful representation to use when considering probabilities is the tree diagram. Tree diagrams provide an organized view of the probabilities of events in a sample space, like this one from the Mathematics Assessment Project's *Medical Testing*. As students gain more experiences calculating probabilities and learn the Addition and Multiplication rules of probability in **NC.M2.S-CP7&8,** consistent use of tree diagrams with simple examples as well as more complex problems will support student success.



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SUGGESTED CITATION

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