NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC^{2}ML Research-Practice Brief #13

NC Math 3 – Functions and Inverses

Function is a conceptual category within the NC High School Mathematics Standards. Prior to revisions, confusion existed around which function families were addressed within each course and some standards appeared in multiple courses. The standards, the Mathematics Resources for Instruction, and the Revisions documents provide clarity and outline the progression of the study of function families across the NC Math 1-3 courses**.**

NC Math 1: Linear, Quadratic, & Exponential
NC Math 2: Quadratic, Square Root, & Inverse Variation
NC Math 3: Exponential, Logarithmic, Polynomial, Rational, Absolute Value, & Trigonometric

The *Function* concept is a vital mathematical concept that is distinguished from "expression", "equation", "formula" and other mathematical relations. A relation that is a *function* must be

> *A relation that pairs every element in one set, called the domain, with exactly one element of a second set, called the range.*

Always relating an input element to its only output element is the predictive, pattern-like power of functions that make them so useful within mathematics and beyond.

ALGEBRA AND FUNCTION ACROSS K-8

Algebra and *Function* are not separate domains in K-5, as students begin to express patterns and generalizations with variables and graph ordered pairs. Students use variables to express mathematical relationships, and progress toward utilizing equations in which variables represent quantities that vary in relation to one another.

In grades 6 and 7 pattern generalization continues as students investigate ratios and proportions, and grade 8 is when students are introduced to a definition of function. In 8th grade, students experience how functions describe relationships between quantities using words, variables, and graphs; and particular attention is given to linear relationships and the change in one quantity with respect to another quantity.

ALGEBRA AND FUNCTION ACROSS 9-12

The use of variables to describe functional relationships keeps *Algebra* and *Function* connected throughout grades 9-12. Algebra can be related to Function as a tool for better understanding characteristics of functional relationships, when those relationships are expressed symbolically. 8th grade students focus primarily on linear functions, their graphical representations, and their graphical characteristics highlighted within the form $y = mx+b$.

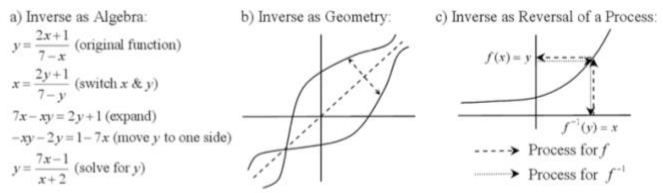
Function notation is not explicit in the standards until NC Math 1. Though the notation is familiar to teachers and ubiquitous across mathematics curricula, "y = f(x)" is a single mathematical sentence that packs a lot of meaning. This sentence shows that "y" and "f(x)" are the same value. It names a function "f", with a generalized input variable "x", and a function value or output "f(x)".

In NC Math 2 students extend their understanding of function to transformations as functions, with two-dimensional domain and range elements within NC.M2.F-IF (think **f(x,y)**, instead of **f(x)**). Here students view a collection of ordered pairs that create an object in the plane, a pre-image, as input values that can be entered into a function to produce the transformed, output image.

NC MATH 3 – UNIT 1, FUNCTIONS AND INVERSES

The NC Math 3 Unit 1 experience of connecting multiple representations of functions (verbal descriptions, numerical values, formulas, and graphs) is a continuation of students' experiences that begin in 8th grade.

In NC Math 3, students grow their collection of function families from NC Math 2 to include polynomials, rational, absolute values, trigonometric, and logarithmic functions. Also unique to NC Math 3 are piecewise defined functions and the notion of inverse functions (NC.M3.F-BF.4). There are three essential understandings to the concept of *inverse function*. First is the "doing and undoing" nature of the operations that define a function and its inverse that can be determined via algebraic manipulation (e.g. $f(g(x))=x$ and $g(f(x))=x$). The second is the graphical reflection of a function's graph over the line y = x to produce the graph of the inverse, so that if (a,b) is a point on the graph of function f, then (b,a) is a point on the graph of f^{-1}. This connects to the third understanding that the domain of a function is the range of its inverse, and vice versa.



Approaches to Engaging with Inverse Functions

This unit integrates *Algebra* and *Function* as students connect formulaic and graphical representations of functions, focusing on how important features appear both graphically and formulaically (NC.M3.A-CED.2, NC.M3.F-IF.7 & 9, and NC.M3.F-BF.1, 3, & 4). Students will connect existing function representations to contexts, build functions from contexts, build equations and inequalities, and use these objects to solve problems (NC.M3.A-SSE.1, NC.M3.A-CED.1, NC.M3.F-IF.2 & 4).

FOCUSING ON A BROADER PERSPECTIVE

A finer notion of *inverse* includes that fact that every function has an *inverse* (operations can be undone and any graph can be reflected about the graph of y=x), but not every function has an *inverse function*, that is an inverse that also satisfies the definition of a function.

It is important to note that in mathematics, calling $\boldsymbol{f^{-1}}$ the *inverse function* of $\boldsymbol{f}$ is equivalent to the relationship of composition in which

$$(f \circ f^{-1})(x) = x = (f^{-1} \circ f)(x).$$

This holds value for teachers as they explore and critique curricular materials involving inverse functions, but it isn't a part of the content explored in NC Math 3. Students in NC Math 1, 2, and 3 do not study the function operation of composition.

STUDENT'S THINKING ABOUT INVERSE FUNCTIONS

As students progress through the NC Math 1, 2, and 3 courses, they build understanding of different function families and their defining characteristics. In NC Math 3, students engage with the idea that families of functions can be related through the concept of inverse functions. For example, the inverse relationship between quadratic and square root functions or exponential and logarithmic functions. In doing so, research has shown that attention to a geometric approach or the relationship between the domain and range of inverse functions can leverage students' understanding of rate of change (Oehrtman et al., 2008). However, research has also shown that students often engage with functions using operational approaches that rely on action (algebraic) rather than relational (graphical and numeric) approaches that rely on process and object understandings (Breidenbach et al, 1992).

QUESTIONS TO CONSIDER

- *In what ways can using an operational approach to teaching inverse functions hinder student's conceptual understanding of inverse funcitons?*
- *How could an attention to other approaches support them?*
- *How can you design appropriate interventions or tasks to advance or refine their conceptions?*

References

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational Studies in Mathematics, 23*, 247-285.

Oehrtman, M. C., Carlson, M. P., & Thompson, P. W. (2008). Foundational reasoning abilities that promote coherence in students' understandings of function. In M. P. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and practice in undergraduate mathematics*, MAA Notes (Vol. 73, pp. 27-42). Washington, DC: Mathematical Association of America.

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NC²ML MATHEMATICS ONLINE

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SUGGESTED CITATION

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