NC COLLABORATIVE FOR MATHEMATICS LEARNING

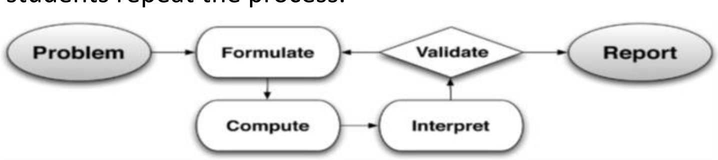
NC²ML Research-Practice Brief #16

NC Math 3 – Modeling with Geometry

Modeling in the NC Standards holds an interesting position, as it is both one of the Standards for Mathematical Practice and identified as particular content in the HS Standards. For example, in NC Math 1 and NC Math 2 modeling makes an appearance in F-LE.1-5, A-CED.1-3, and S-ID.7-9. However, the only place that an entire unit focused on mathematical modeling appears in the NC HS Collaborative Instructional Framework is in this Modeling with Geometry unit in NC Math 3. Students are asked to apply known geometric relationships and formulas to model with mathematics and to solve problems.

WHAT IS MATHEMATICAL MODELING?

Mathematical modeling refers to using mathematical approaches to understand and make decisions about real-world phenomena (GAIMME, 2016; CCSSM, 2010). The modeling process both starts and ends in a real-world context. This first phase requires that students identify variables, essential features of the context, and make assumptions to narrow the messiness of the problem. In the next phase, formulate, students mathematize the problem by creating and selecting geometric, algebraic, or statistical representations that describe the relationships between the variables. What they create in this phase is the ***mathematical model***. They then work mathematically (compute) to find a solution, interpret the solution in the context of the problem, and validate the solution. In the validation phase, one of the questions to be considered is whether or not the model is the best it can be. If the answer is, “not yet”, then revisions to the model are made and students repeat the process.



The Modeling Process (CCSSM, 2010)

Finally, when students are satisfied with their model a report is produced that communicates all of the decisions made, the model itself, and the real-world solution based on this model.

Context is what really sets a modeling problem apart from a typical “word problem”. The context is authentic and messy and the mathematical work is integrally connected to the context (Tran and Dougherty, 2014). Before students even begin formulating a model, they must build an understanding of the situation. Context might take a backseat to the mathematical work of calculations as students move through the modeling process, but it returns to the forefront as students report their models and solutions (McCulloch et al, 2017). The table to the right clarifies what modeling is and isn’t.

Modeling is…	Modeling is not…
Writing equations to determine whether or not drones or delivery trucks are most cost effective for particular delivery points.	Simply writing and solving equations for how many apples and oranges Jen bought at the farmers’ market.
Creating a representation to estimate how much water is needed to provide families in Flint, Michigan, what they need per household for a month.	Just using manipulatives, although manipulatives can help in the problem-solving process.
Using real data to determine the best location of a solar panel parking structure for a large national chain store.	Creating a graph just because the problem says to create a graph.

McCulloch, Whitehead, Lovett, & Whitely (2017) p.

MODELING CIRCLES IN NC MATH 3

There are few *specific* contexts for modeling that you must do in this unit. One is a context in which students ultimately model the path of a circle (as a conic section) by deriving the equation of the circle using the Pythagorean Theorem (**NC.M3.G-GPE.1**). There are endless possibilities of situations that could be posed in which this model (i.e., the equation of a circle) would emerge. For example, you might pose the following problem:

Bob is going to be driving a moving truck to his new home in North Carolina. On his drive, he will be traveling on a two-lane road through the Blue Ridge Mountains that has a semicircle shaped tunnel. Does he have reason to worry about driving the moving truck through the tunnel?

Students will need to make assumptions about the measures of the truck and tunnel, decide their variables, and then using what they know (the Pythagorean Theorem) will create a mathematical model of the path of the tunnel (equation of a circle) and use it to test whether or not their truck can fit.

HOW DO YOUR STUDENTS DEFINE A CIRCLE?

Tie a short string to your pencil. Then holding the other end of the string still, sketch the circle of points that are the string's distance from the center.

If you ask students to draw 3 different circles on their paper, what characteristics could be used to distinguish one circle from another?

Before now, students may or may not have thought of this geometric object as the collection of all points equidistant from the center. Using a modeling task like the one above creates opportunities for students to visualize this definition of a circle as they derive the equation of a circle with center (0,0). Students can then use transformations to create the equation of a circle centered at any point, (h,k). Students are also expected to complete the square to write the equation in center-radius form so that the center and radius are identifiable.

MODELING 3D OBJECTS IN NC MATH 3

NC.M3.G-GMD.3&4 call for students to explain volume formulas and to visualize relationships between 2D and 3D objects. The 2D figures may be the surfaces that make up the faces of the 3D objects, the results of cutting cross-sections of the 3D objects, or what is rotated to create the 3D object. As students connect these 2D parts to their 3D whole, these relationships may help bring meaning to the structure of the formulas for volume. For example, a cylinder is the result of stacking an area of πr^2 on itself h times and a prism of any base-shape has a volume formula that fits this stacking notion. It is also important to consider why this same stacking reasoning doesn't work for thinking about the volume of a cone or pyramid.

The visualization within this unit will be put to use as students apply their knowledge of area and volume formulas to modeling situations. Students could attempt to calculate the thickness of a soda can (see Illustrative Mathematics) or use a diagram of the cross section of a nail to calculate its volume (see Mathematics Vision Project, Task 5.4:"Hard as Nails"). Visualizing the solid and its cross-sections determines the appropriate formulas needed to calculate a volume or an area.

You might also draw on Dan Meyers collection of Three-Act Tasks within this unit. These tasks draw on storytelling to engage students in mathematical modeling. The Meatballs task starts with a video of meatballs being placed into a pot of simmering tomato sauce. Upon watching the video, a natural question emerges - How many meatballs will it take to overflow? Asking students to explore this situation will have them drawing on their knowledge of volume of spheres and cylinders as they come up with their varied answers to that question.

Mathematical modeling provides opportunities for students to draw on mathematics they know as well as create need for new mathematics (as we see in the tunnel task) in the context of doing the kind of "real" mathematical work. While mathematical modeling is central to this particular unit, opportunities to model should be included whenever appropriate and possible.

References

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SUGGESTED CITATION

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