NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC^{2}ML Research-Practice Brief #17

NC Math 3 - Reasoning with Geometry

A ***mathematical proof*** is a coherent argument built of mathematical facts and relationships that support a conclusion. When students are asked to ***prove*** theorems in mathematics, it is often the case that the expected product is more sophisticated then what students might create when asked to ***verify***, ***apply***, ***demonstrate***, ***explain***, or ***justify***.
It's important to clearly communicate such expectations with students, if we intend for a ***proof*** to be distinct from an ***answer*** or a ***solution***.

Instruction in geometry has traditionally housed a student's first encounter with the directive ***prove***. This remains true in our current state content standards, though our students will be well prepared for the task of proving since the practice standards ensure that students will be "constructing viable arguments" prior to their first encounter with ***proof*** (which will occur in NC Math 1 – NC.M1.G-GPE.5; where geometry in the plane connects to the algebra of linear equations).

NC MATH 3 – UNIT 5, REASONING WITH GEOMETRY

When students reason with geometry, they utilize *given geometric characteristics* to explain relationships within and between geometric objects. The objects of focus in this **5th Unit** of the Collaborative Pacing Guide for NC Math 3 include but are not limited to triangles, parallelograms, and circles, building on the exploration of triangles and parallel lines that students engage with in NC Math 2.

More specifically, students in NC Math 3 will draw upon relationships related to triangle congruency theorems and theorems about lines and angles (M2.G-CO.9,10) learned in NC Math 2 to prove and apply geometric theorems about triangles (M3.G-CO.10), parallelograms (M3.G-CO.11; G-C.5), and circles (M3.G-C.2) to solve problems. Additionally, they will draw upon their understanding of the Pythagorean Theorem (8.G.8) and completing the square for quadratic expressions (M2.A-SSE.3) to explore the equation of a circle on the coordinate plane (M3.G-GPE.1).

THE IMPORTANCE OF PROOF & STUDENT REASONING

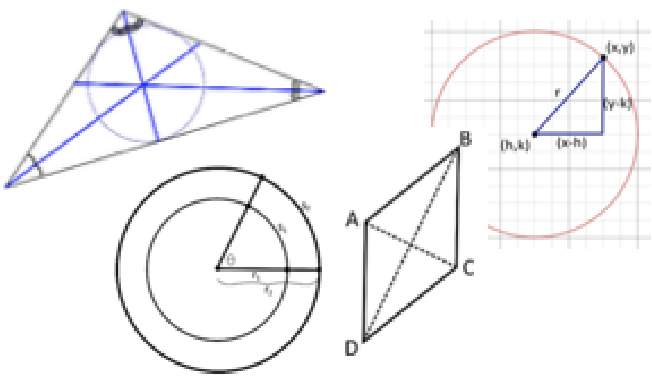
All too often, proof is treated as separate from the rest of mathematics; however, proof is central to the work of mathematicians and should be an essential component of mathematics instruction. Supporting this notion, researchers have outlined several important ideas in considering the importance of proof (Ellis et al., 2012). For example, a proof demonstrates truth for all possible cases and thus is not an argument based on intuition or mere examples. However, research has shown that students often view proofs as a proof of only the single case of the problem they are proving and thus do not hold strong conceptions that "guarantee safety from counterexamples" (Chazan, 1993, p. 382).

Additionally, the work of proving can provide support for the development of precise mathematical language and thus clear communication of mathematical knowledge, both of which are important in explaining and understanding mathematics (Ellis, et al., 2012). Research has shown that providing occasions for students to compare ideas gives them opportunities to develop their language as they modify, consolidate, strengthen, or reject arguments in attempts to develop proofs (Maher & Martino, 1996).

FROM CONJECTURE TO PROOF - AN EXAMPLE

Given the importance of proof and students' potential conceptions, students need time to conjecture, investigate, and engage in geometric problem solving prior to being able to piece together a logically structured collection of premises that build a proof.

Consider the importance of determining if two different definitions are equivalent or if starting with different *given geometric characteristics* leads to different conjectures.



When facilitating students' opportunities to solve geometric problems and reason geometrically, teachers must anticipate how students might use the "*given geometric characteristics*" to conjecture about other characteristics that might be true. Further, teachers will need to provide tools for students to utilize when investigating geometric relationships, keeping in mind that different tools may better highlight different characteristics.

For example, if students begin with the definition,

> *A* ***parallelogram*** *is a quadrilateral with opposite sides that are parallel,*

then they are starting with *given geomteric characteristics*. Students could then conjecture about what other geometric characteristics they think must be true of a parallelogram.

Building on the NC Math 2 knowledge, the conjecture that *opposite angles of a parallelogram* are congruent may be a starting place to build an accessible proof that relies on the definition of a parallelogram, congruency of angles created from a transversal, and angle-side-angle triangle congruency.

If given a piece of paper with a parallelogram, students may: fold the paper to justify relationships they see (e.g. opposite angles, opposite sides, bisecting); use scissors to justify similar or additional relationships (e.g. area as the product of base and height, consecutive angles are supplementary); or use geometric software (e.g. www.geogebra.org; www.desmos.com/geometry) to examine side lengths, diagonal lengths, and angle measures to verify relationships.

BUILDING GEOMETRIC HABITS OF MIND

Given the importance of proof and students' potential conceptions, students need time to conjecture, investigate, and engage in geometric problem solving prior to being able to piece together a logically structured collection of premises that build a proof. In addition, teachers need to support students in distinguishing differences between a non-proof, a rationale, and a proof (Ellis et al., 2012). In brief, a *non-proof* provides only examples; a *rationale* explain how but not why, may use logical reasoning, and/or drawings; and a *proof* explains truth for all cases, uses deductive reasoning, and does not use examples.

To support students, Driscoll et al. (2007) offers a *geometric habits of mind framework* to as a guide for the kinds of tasks and questioning teachers can engage in with their students in order to develop students' geometric thinking. Instruction should provide students with opportunities to:

- ***look for relationships*** within and between figures;
- ***generalize geomtric ideas***;
- ***investigate invariants*** (things that stay the same); and
- ***provide equal opportunities for both exploration and reflection.***

QUESTIONS TO CONSIDER

- *How does a students' understanding of proof influence the ways they engage with geometry tasks?*
- *How can you structure lessons to support students in moving from non-proof to rationale to proof?*
- *How can you support your students in developing stronger geometric habits of mind?*

References

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SUGGESTED CITATION

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