NC Math 3 - Rational Functions

Rational functions are ratios of polynomials (with non-zero denominators), as **rational numbers** are ratios of integers (again, with non-zero denominators). The [NC Instructional Framework] places this unit in NC Math 3 after two geometry units. This functions unit connects back to the work done in Unit 3 with polynomials, since polynomials are the building blocks for rational functions.

MODELING WITH RATIONAL FUNCITONS

Rational functions are important for NC Math 3 students to understand because of their usefulness in modeling physical and financial phenomena. One example is Young's rule for pediatric (under age 12) dosing, $Y(x) = \frac{ax}{x+12}$ where a is the adult dosage and x is the child's dosage (Bedgood, 2008). An example of a financial model is the function $C(n) = \frac{P+kn}{n}$ *for some P and k,* which models the annual cost of a product that is owned for *n* years. The last one might be of particular interest for many high school students!

An important part of modeling with mathematics is to consider the strengths and weaknesses of the mathematical formulas used within the model, and to connect how the formulas used reflect what would really happen. For example, when considering the given *f(t)*, the function accurately represents how much medication would be in the bloodstream before any dose is taken (at time zero).

CONNECTING TO STUDENTS' PRIOR UNDERSTANDING OF RATIONAL NUMBER

The NC Instructional Framework suggests that this unit begin with a focus on rational expressions, building off of students' prior understanding of rational number. Further connections to number and operations are made within the content of **NC.M3.A-APR.6 & 7**, as NC Math 3 teachers find themselves in a situation that is similar to one faced by their 4th grade teacher colleagues… to teach a division algorithm or not. That's a great question!

See the [NC Math 3 Unit 3 Brief] for more supporting the role of ***sense-making*** for students prior to exposure to polynomial or synthetic division. The standards of this unit call for students to be able to divide polynomial expressions, but the successful application of any algorithm is supported by students understanding the relationships between the quantities involved. Much like the importance of developing number sense related to rational number, research has shown that students with strong symbol sense work more flexibly with rational expressions than those that do not (Arcavi, 1994). Symbol sense is described as "a quick or accurate appreciation, understanding, or instinct regarding algebraic symbols" (Arcavi, 1994, p.31). Rational expressions are composed of many familiar algebraic symbols, but seeing them all together can be overwhelming for many students. That is why it is so important to connect them to their uses in other functions and to rational number. Some great questions to pose to your students in support of this goal are in the box below.

DISCUSS WITH YOUR STUDENTS

- *How could you use equivalencies to* $\frac{9}{8}, \frac{7}{8}, and \frac{9}{9}$ *to make sense of* $\frac{x+1}{x}, \frac{x-1}{x}, and \frac{x^2+1}{x^2+1}$ *?*
- *Before we divide, which of these quotients should be greater than 1? How do you know?*

$$\frac{x+2}{x-1} \quad \frac{x^2-1}{x^2} \quad \frac{x^2+4x+4}{(x+2)^2} \quad \frac{x^3-1}{x-1}$$

- *Knowing that* $x^2 - x - 6 = (x-3)(x+2)$*, what is the quotient of* $\frac{x^2-x-6}{x+2}$ *?*
- *Since* $\frac{2x^2-4}{x-1} = 2x+2,$ *then the product* $(x-1)(2x+2)$ *would be equal to _________.*
- *How does* $\frac{6}{5} = 1 + \frac{1}{5}$ *relate to* $\frac{x+1}{x} = 1 + \frac{1}{x}$ *?*
- *How do* $\frac{2x+2}{x}$ and $2 + \frac{2}{x}$ and $2\left(1 + \frac{1}{x}\right)$ *relate?*
- *Why doesn't* $\frac{x^3+x}{x+3} = x^2 + 1$*?*

Finding ways to motivate the need for rational expression operations and simplification is not always easy. One suggested strategy (Meyer, 2015) is putting students in a situation in which the simplified form is far more efficient than its equivalent counterpart. Not only does this create a need for methods of simplification, but it highlights the equivalence of the two rational expressions (**NC.M3.A-SSE.3c, NC.M3.A-APR.6**).

CONNECTING TO STUDENTS' PRIOR UNDERSTANDING OF FUNCTION

As with other function families in this course **rational functions** are analyzed, in part, through their graphical representations (**NC.M3.F-IF.4, 7, & 9**). Students will extend their study of function characteristics (domain, range, intervals of increase or decrease, intercepts, etc.) to include discontinuities due to asymptotes. Using a graphing utility is a great way to efficiently create multiple graphs so that students can look for patterns in order to generalize shape, including the location of vertical asymptotes. The online calculator Desmos hosts a Teacher Desmos site offering lessons that are classroom ready (visit the Teacher Desmos site and search for *rational functions ; one such activity is the Polygraph-Rational Function Activity).* Other investigations are made easier with technology since students can quickly check graphs of functions. For example, evidence of student understanding of function characteristics can be elicited through designer function types of tasks.

SAMPLE DESIGNER FUNCTION TASKS

Designer function tasks ask students to design functions that meet given criteria. Two examples are below.

- *Can you create a rational function with 4 vertical asymptotes and 3 relative extreme points?*
- *Can you create two different rational functions, each with a vertical asymptote at x=3 and a zero at x=-2?*

In order for students to gain a deep understanding of functions, they need to see the connection between algebraic and graphical representations. Kop et al. (2015) discusses the importance of being able to visualize the graph of a function when presented with an algebraic representation. Although the research about linking multiple representations is appropriate to all function types, it is especially relevant to rational functions. Students who successfully create a mental picture of a rational function graph along with its subtle characteristics (i.e., domain, range, asymptotes, end behavior) when presented with an algebraic representation can tailor their general graph to fit the function (Kop et al., 2015). For example, given $f(x) = \frac{1}{x-2}$, students can use the mental image of the parent graph $f(x) = \frac{1}{x}$ as a frame of reference that they can then translate right two units to the right. Students know that the domain of the parent function is discontinuous $(x \neq 0)$, thus the domain of $f(x)$ must follow suit.

A wonderful way to assess student understanding of rational functions is through the creation of what are often called designer functions. These are functions that have been designed to meet a particular set of constraints.

DISCUSS WITH YOUR COLLEAGUES

- *What instructional decisions can you make in this unit to ensure students develop both conceptual understanding of rational functions and their equivalent forms, as well as procedural fluency related to operations with them?*
- *How might you motivate the need for rational expressions/functions, simplifying rational expressions/functions, and operations with rational expressions/functions?*
- *Which previous function families will be particularly helpful to compare rational functions to when highlighting characteristics that define rational functions? Explain your choices.*
- *Consider the affordances and constraints of different representations of rational functions (e.g., graphs, symbolic, tables of ordered pairs). Look at each standard in this unit and discuss which representations you think will be important to draw upon in relation to each.*

References

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