NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC^{2}ML Research-Practice Brief #19

NC Math 3 – Trigonometric Functions

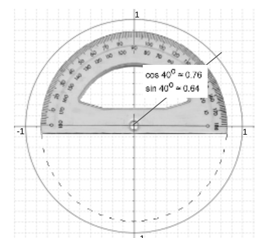
NC MATH 3 – UNIT 7, TRIGONOMETRIC FUNCTIONS

The NC Collaborative Pacing Guide for NC Math 3 places Trigonometric Functions as the seventh unit, following Reasoning with Geometry (in Unit 5). This order is important because ***radians*** are introduced in the *geometric* study of circles in Unit 5, then utilized in the *trigonometric* study of Unit 7. (See The Mathematics Resource for Instruction for more details.)

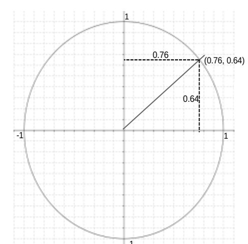
In NC Math 2, students are first introduced to trigonometry by engaging with ***sine***, ***cosine***, and ***tangent*** as ratios of side lengths in right triangles. These definitions are applied to contextual problems and proofs about triangles. While the study of *special right triangles* in NC Math 2 provides students experience with ***trigonometric benchmark values*** *(i.e., sine, cosine, and tangent of 30, 45, and 60 degree angles)*, the focus is only on trigonometric values of acute angles. In NC Math 3 this is expanded to the study of ***sine*** and ***cosine*** as ***functions*** with domains that can be related to the set of all possible angle measures, while working with the units of both ***radians*** and ***degrees*** and connecting to the representation of the unit circle (NC.M3.F-TF.1,2).

TRIGONOMETRIC FUNCTIONS AS OPERATIONS

Research has shown that students who hold a solely ratio view (i.e., right triangle trig) of trigonometric functions struggle to develop a deep understanding of the functions, are unable to justify properties of these functions, and have difficulty estimating outputs of the functions (Moore, 2014; Weber, 2005). One of the reasons that trigonometric functions are difficult for many students is that it is the first time they see a function in which the actions to take when evaluating the function may not be clear. For example, when one sees a square root or a cubic function, students know what actions to take when evaluating the function for a given number. Trigonometric functions can be thought of as operations on angles. For example, sine can be thought of as the process



of using a protractor to draw a ray emanating from the origin of the unit circle, locating the point of intersection of the ray and the unit circle and determining the y-value of that intersection. Weber (2005) found that students who experienced sine, cosine and tangent functions in this way were able to not only estimate trigonometric functions at any angle, but also developed a deep understanding of the properties of trigonometric functions and their graphs. (See Weber, 2008 for a task that addresses this). This connection between right triangle trig and the unit circle is key to helping students bridge the connections between the ratio view of trigonometry and trigonometric functions as is expected in NC Math 3. See NC.M3.F-TF.2 in the MRI Document for more information on making these connections and the depth needed in NC Math 3.



THE UNIT CIRCLE

The unit circle is one of the important big ideas in NC Math 3. However, it is important to note that students are not expected to memorize it. Rather, it is important that they develop a conceptual understanding of how the sine and cosine values change as angles of rays drawn within the unit circle change. One way to do this is to draw on their understanding of geometric transformations and right triangle trig ratios (using rays to identify the right triangles). Using the special right tirangles (trigonometric benchmarks) and reflecting them around the unit circle (NC.M3.F-TF.2) students should be asked to recognize patterns that exist within the unit circle and be able to construct the sine and cosine graphs from it. For example, using knowledge of trigonometric benchmarks, students can approximate the sine of 40° as:

$$\sin 30 < \sin 40 < \sin 60 \textbf{ THUS } \frac{1}{2} < \sin 40 < \frac{\sqrt{2}}{2}.$$

Additionally, students might notice that the sine and cosine values for the trigonometric benchmarks have common outputs of $\frac{1}{2}, \frac{\sqrt{2}}{2}$, and $\frac{\sqrt{3}}{2}$. This is true because of the complementary relationship held by the pair of acute angles in right triangles. This realization builds on student knowledge of trigonometric ratios from NC Math 2.

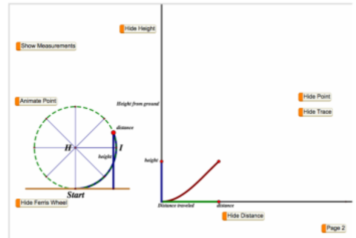
A great way for students to build this understanding is by placing cut-out, *special right triangles* (30-60-90, 45-45-90) with a hypotenuse of length 1 on the unit circle, marking radii at degree-values by reflecting the triangle across the axes in a counterclockwise order. Thus, using the triangles to draw radii at 30°, 150°, 210°, and 330°, then 60°, 120°, 240°, and 300°, then finally 45°, 135°, 215°, and 315°.

GRAPHS OF TRIGONOMETRIC FUNCTIONS

Building on prior knowledge of rigid transformations from NC Math 2, tasks that require students to reflect right triangles within the unit circle like described above provide opportunities for students to realize that although reflecting the special right triangles changes their orientation, the images and pre-images are congruent. More specifically, students focus their attention on the location of the radial endpoint in the coordinate plane of each triangle to see why ***sin(30°)=sin(150°)*** and ***cos(30°)=-cos(150°)***. This allows students to connect their understanding of trig function outputs to ***xy***-coordinate locations and utilizing ***benchmark values*** to determine other outputs of the functions.

A great task to connect ***trigonometric ratios***, ***unit circle coordinates***, and the ***graphs of trigonometric functions*** Is the *Graphs from the Unit Circle* which comes from the *National Council of Teachers of Mathematics* (NCTM). In this task students use the horizontal and vertical distances to points on the unit circle to create the graphs of the ***sine*** and ***cosine*** functions, connecting back to Weber's suggestions for introducing trigonometric functions as operations on angles to identify the xy-coordinate locations.

Once ***trigonometric*** relationships have been graphically connected to the concept of ***function*** students will be able to investigate the function characteristics they have already considered for polynomial, power, and exponential functions (NC.M3.F-IF1, 4, 7, & 9). In this unit they will be adding ***periodicity*** to their list of function characteristics to study. In NC Math 3, students are asked to examine the parameters of a, b, and h in terms of the function $f(x) = a \cdot \sin(bx) + h$ (NC.M3.F-TF.5). The MRI stresses to not overreach with this standard. NC Math 3 provides students an opportunity to *begin* building the concepts of the sine function and the effects of the various representations by changing parameters. Students should definitely be ***using technology to investigate* these changes.** An example of this is the use of the Ferris Wheel simulation. Students examine dynamic images of a Ferris wheel and attend to the height of a rider from the ground while the ride is in motion. Research has found that students that engaged in modeling



this dynamic situation were able to not only describe and graph the motion using ***trigonometric functions***, but also were able to justify the curvature in their graphs by drawing on their understaing of the relationships between right triangle trig and the unit circle. Specifically, students who used relationships between time and height to make sense of the situation could move flexibly between ***covariation*** and ***correspondence*** perspectives of function, thus not only deepening their understanding of this particular trigonometric function, but functions more generally (Johnson, Hornbein, & Azeem, 2017). There are excellent examples of dynamic Ferris Wheel tasks at Desmos Teacher, Illuminations, and WebSketchPad.

Questions to consider…

- *What are the advantages and disadvantages to separating "right triangle trig" from work with trigonometry functions?*
- *Why is it important to connect trig functions in NC Math 3 back to the work with the ratio perspective that students worked with in NC Math 2?*

References

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Weber, K., Knott, L., & Evitts, T. (2008). Teaching trigonometric functions: Lessons learned from research. Mathematics Teacher, 102(2), 144-150.

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NC²ML (2018, February). NCM3.7 Trigonometric Functions. *Research-Practice Briefs*. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. Retrieved from nc2ml.org/brief-19