NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC²ML Research-Practice Brief #9

NC Math 2 - Quadratics

NC MATH 2 – UNIT 3, QUADRATICS

The third unit in NC Math 2, **Quadratics**, is all about quadratics! In NC Math 1, students begin to build an understanding of quadratics by examining key features and coefficients to factor and solve quadratic equations in context. In NC Math 2, the NC Collaborative Pacing Guide Unit 2 standards extend this work to focus on: *creating* equations describing quadratic relationships; *analyzing* and *interpreting* equations by making connections across representations; *comparing* and *transforming* quadratic relationships, and an introduction to complex numbers.

The distinction between Quadratics within the two courses is not in their blend of *Algebra* and *Function*. Both sets of course standards call for connecting algebraic and graphical representations. Instead, the algebraic maneuvering becomes more sophisticated in NC Math 2. In NC Math 1, students rewrite "factorable" quadratic expressions and solve for real roots, considering them as zeros to the corresponding functions. In NC Math 2 students engage with more sophisticated algebraic techniques including *completing the square* and *the quadratic formula*, while continuing to be able to justify their work.

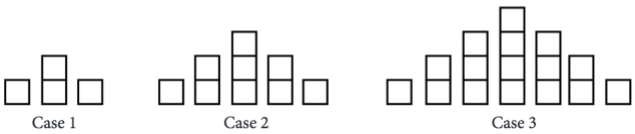
NC.M2.A-REI.4: Understand that the quadratic formula is the generalization of solving $ax^2 + bx + c$ by using the process of completing the square.

NC.M2.A-SSE.3: Write the equivalent form of a quadratic expression by completing the square, where a, is an integer of a quadratic expression, $ax^2 + bx + c$, to reveal the maximum or minimum value of the function the expression defines.

BUILDING FROM NC MATH 1

When deciding how to begin this unit, it may be useful to start with a task that is accessible using approaches that build upon mathematics learning in NC Math 1. Growing pattern tasks in which students must identify a pattern, generalize the pattern, and graph the relationship can be utilized to address numerous standards within this unit (e.g. NC.M2.A.SSE.1; F.BF.1; A.CED.1; A.CED.2) and can help build upon students growing understanding of rate of change and function families.

How do you see the shapes growing?



https://www.youcubed.org/task/squares-upon-squares/

In NC Math 1, students engage first with linear and exponential functions, building an understanding of their respective additive and multiplicative rates of change. That is, linear functions can be characterized by a constant rate of change, and exponential functions by a rate of change that is proportional to the value of the function. While not explicit in the standards, many advocate that teachers can support students in extending their understanding of quadratics by designing instructional opportunities that highlight that quadratics have a linear rate of change (NCTM, 2014). For example, in the growing shapes problem above, by examining different representations, students may recognize that the pattern has a rate of change (or first difference) that changes linearly and a second difference that is constant.

STUDENT'S THINKING ABOUT QUADRATICS

Extending from the growing shapes example, research has shown that students are able to recognize the average rate of change over equal intervals of a quadratic function as linear (Lobato et al., 2012). Similarly, they are also able to recognize symmetry, however they may struggle in making sense of symmetry conceptually and in understanding the relations between quadratic expressions, equations, and functions (Nielson, 2015).

COMPLETING THE SQUARE

Another point of instructional decision-making will arise when introducing the algebraic technique of *completing the square*. Well-designed curricular materials highlighting a visual approach can be used to show students that this process can actually **complete** a **square**, while maintaining the original equality. This understanding can be cultivated by using algebra tiles (either real or virtual). A visual representation of this algebraic process can bring meaning to the symbolic manipulation and to why new expressions are equivalent to the original. Such experiences can support students in successfully implementing the procedure.

QUADRATIC FORMULA

When introducing the *quadratic formula*, drawing from standard ***NC.M2.A-REI.4***, teachers could decide to derive the formula from a quadratic polynomial written in standard form. Beginning with

$$ax^2 + bx + c = 0$$

subtracting ***c*** from both sides of the equation and dividing the equation by the factor ***a*** results in

$$x^2 + \frac{xb}{a} = -\frac{c}{a}$$

Applying the procedure of completing the square to this equation, attending to maintaining equality, and then solving for **x** results in the quadratic formula.

The goal of sharing such a derivation is not to have students memorize steps, but rather to connect what students know to this new relationship. Making sense of ***why*** the quadratic formula works helps to ensure students understand when to use it and to use it effectively. Since every NC Math 2 class of students is unique, it is up to teachers to decide how to best utilize this derivation (lecture, investigation, whole group line-by-line discussion, think-pair-share, etc.), in order to maximize students' understanding.

SOLUTIONS – ROOTS, ZEROS, & COMPLEX NUMBERS

A *root* of a polynomial is a value that makes the polynomial expression equal to 0. A *zero* of a polynomial function is the input that outputs a function value of 0, thus a zero of a function is the x-coordinate of an x-intercept on the graph of the function. Very similar, yet not exactly the same thing!

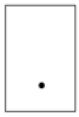
Every quadratic expression has at most two roots. But it's not the case that every quadratic function has zeros. As students investigate a variety of quadratic functions, they will be introduced to a new set of numbers, the set of complex numbers. Built from the set of real numbers using the imaginary unit ***i***, complex numbers provide solutions to equations of the form $\mathbf{ax^2+bx+c=0}$ that do not have real number solutions. Students will be calculating solutions, connecting the presence of real or complex solutions to the graphical representation of the quadratic function.

EXTENSION – MATH KNOWLEDGE FOR TEACHING

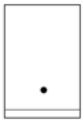
What is your definition of a ***parabola***? Many would say that a ***parabola*** is the name of the shape given to the graph of a quadratic function. A more formal definition would be that a ***parabola*** is the collection of all points in the plane equidistant from a given point, the *focus*, and a given line, the *directrix*. The shape is one of four called *conic sections*, since it can be created by intersecting a cone with a plane.

Use a Focus and a Directrix

- *Mark a single point on a piece of paper, about 3 inches from the bottom and centered. (Patty paper works best.)*



- *Draw a horizontal line across the paper about 2 inches below the point and place at least 10 tick marks on the line across the paper.*



- *Fold the paper so that one of the tick marks overlaps the point and make a crease along the fold.*
- *Trace the line that is made by the crease with a pencil.*
- *Repeat this process for each tick mark.*

Questions to consider with your colleagues:

- What do you notice?
- Considering the ways in which you have taught this unit in the past, what is one thing you would like to do differently to build on your students conceptions of quadratics?

References

Lobato, J., Hohensee, C., Rhodehamel, B., & Diamond, J. (2012). Using student reasoning to inform the development of conceptual learning goals: The case of quadratic functions. *Mathematical Thinking and Learning*, *14*(2), 85-119.

Nielsen, L. (2015). *Understanding quadratic functions and solving quadratic equations: an analysis of student thinking and reasoning* (Unpublished doctoral dissertation). University of Washington, Seattle, WA.

National Council of Teachers of Mathematics (NCTM). (2014). *Putting Essential Understanding of Functions into Practice 9-12*. Reston, VA: Author.

LEARN MORE

Join us as we journey together to support teachers and leaders in implementing mathematics instruction that meets needs of North Carolina students.

NC²ML MATHEMATICS ONLINE

For more information on accessing Canvas learning modules or additional resources please visit http://nc2ml.org/

SUGGESTED CITATION

NC²ML (2018, February). NCM2.3 Quadratics. *Research-Practice Briefs*. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. Retrieved from nc2ml.org/brief-9