# Middle School Mathematics
# Collaborative Instructional Framework

The following Collaborative Instructional Framework is meant to serve as a guide for teachers and districts as they organize the curriculum for the school year. Unlike traditional pacing guides, the instructional framework consists of clusters of standards that are meant to be adapted to various schools and contexts. The instructional framework used research on students' learning progression in mathematics to create and order clusters of standards that are taught together. While there is a strongly suggested order for teaching the clusters, we recognize that schools differ in their contexts and may wish to switch the order around. In those cases, we have given guidance regarding alternative clusterings; however, we note when certain clusters need to be taught in a certain order.

The Collaborative Instructional Framework was created over a five-month period, beginning in July. Twenty individuals from NC DPI, classroom teachers, district leaders, and university faculty worked together to a) read research about pacing guides, student learning progressions, and standards, b) determine the best clusterings per grade level based upon research, when possible, and c) wrote this draft of the framework. The members of this Middle School Framework Team include: Jen Arberg, Lisa Ashe, Stefanie Buckner, Caroline Butler, Chris Cline, Tara Costenoble, Dr. Deborah Crocker, Jill Hooley, Robert Leichner, Kim McCuiston, Dr. Katherine Mawhinney, Dr. Gemma Mojica, Nicolette Morgan, Joseph Reaper, Claudette Reep, Dr. Luke Reinke, Melanie Richey, Audrea Saunders, Patricia Shumaker, and Stacy Wozny. These mathematics professionals represent the four main regions of NC as well as urban, rural, and charter schools. Special thanks to Joseph Reaper and Lisa Ashe from NC DPI for providing guidance and checking for consistency among the framework and DPI resource documents.

CCSS.MATH.PRACTICE.MP1
Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP2
Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP3
Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4
Model with mathematics.

CCSS.MATH.PRACTICE.MP5
Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP6
Attend to precision.

CCSS.MATH.PRACTICE.MP7
Look for and make use of structure.

CCSS.MATH.PRACTICE.MP8
Look for and express regularity in repeated reasoning.

# Standards for Mathematical Practice

The Standards for Mathematical Practice are critical ways of acting and communicating in classrooms that should be instilled in students throughout the school year. Whether students are learning to reason proportionally or statistically, they should be obliged to make sense of the problems posed (MP1) and create a mathematical solution that can contribute to their peers' and their own learning. When solving a problem, such as which company is the cheapest when comparing the prices of t-shirts, students should be able to create a viable argument for their choice, with mathematical evidence to defend their solution (MP3). Students should be able to move among various representations, reasoning quantitatively with symbols (MP2) and creat models of both everyday and mathematical situations they encounter (MP4). Teachers should provide opportunities for students to reason with a variety of tools (MP5), including technologies that are specific to mathematics (e.g., calculators, Desmos, GeoGebra, etc.). Attending to precision (MP6) is a practice in which students attempt to present clear arguments, definitions, and meanings for symbols as they explain their reasoning to others. Finding patterns and structure is crucial throughout the standards as students attempt to mathematize complex problem situations (MP7). Finally, students should attempt to find regularity in reasoning, such as recognizing that the slope is the coefficient of the x term in a linear equation.

# 7th Grade Mathematics Clusters

The clusters are recommended using the progression below, but this is not the only possible progression teachers may use. When considering changes, please look to the "Connections & Rationale" for notes about when one cluster must follow another. The timing for the units in the "Recommended Timeframe" should not be treated as a rigid time table. Also, continue to focus on how the Standards for Mathematical Practice can be incorporated with these content clusters.

| Recommended Order | Alternative Order |
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| Proportional Relationships<br>Reasoning with Rational Numbers<br>Probabilistic Reasoning<br>Reasoning About Expressions<br>Reasoning About Equations and Inequalities<br>Geometric and Measurement Reasoning<br>Reasoning about Population Samples<br>Comparing Populations | Proportional Relationships**<br>Reasoning with Rational Numbers**<br>Probabilistic Reasoning<br>Reasoning About Expressions<br>Reasoning About Equations and Inequalities<br>Geometric and Measurement Reasoning<br>Reasoning about Population Samples++<br>Comparing Populations++<br><br>Clusters with ** can be switched in order<br>Clusters with ++ can be switched in order |

A significant emphasis should be placed on understanding operations with fractions and decimals and proportional reasoning. Anecdotal evidence and teachers' experiences indicate that developing strong proportional reasoning in students is a challenge. As 7th graders are developing new understanding of rational numbers and operations with rational numbers, they are also being asked to reason and solve problems with ratios and proportions. Since "fraction" and "ratio" are not exactly the same thing, teachers need to be aware of the fact that success in operations with fractions does not always indicate an ability to reason with ratios. The mathematics of fractions is often the mathematics of ratio, but the reasoning with each is not the same. One resource for more information about reasoning with ratios and proportions in NCTM's *Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning, 6-8*.

Though there is a single cluster entitled "Proportional Relationships", teachers should not anticipate that teaching this one unit will suffice as the single moment that develops their students' proportional reasoning. Developing students' proportional reasoning, should be a goal throughout the 7th grade year.

Please pay attention to the "Supporting Standards" and "Connections & Rationale" portions of this recommendation. These tools should help to connect mathematical concepts across the units, and to highlight potential opportunities for revisiting previous units' main ideas.

Advice for thinking about any re-ordering of these content clusters:

1. It is recommended that the first week of the school year be spent engaging students with open-ended mathematics problems designed to support the students' growth mindset. This first week is also an opportune time for setting up the classroom expectations and norms for collaborating with classmates and participating in whole class discussions.

   Youcubed.org offers 3 *Weeks of Inspirational Mathematics*. It is recommended that 6th graders experience Week 1, 7th graders Week 2, and 8th graders Week 3. However, other general problem solving lessons can be used.

2. There are numerous ways to connect from one cluster to the previous or next cluster. Look to the standards in order to bridge these units.

3. Note that **Proportional Reasoning** and **Reasoning with Rational Numbers** appear all along the length of the school year. We encourage a continued focus on developing proportional reasoning and understanding/fluency with rational number operations. Though there may be a unit that addresses these topics for a focused period of time, the topics should be integrated throughout the entire year.

| Standards/Cluster | Recommended Timeframe | Supporting Standards | Important Notes |
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| **Jo Boaler's Week of Inspirational Math - Week 2**<br><br>**-Or-**<br><br>**Other problem solving and environment-building activities** | **1 week** | | The intention of the first week(s) of class is to establish a mindset that math is about patterns and struggle is good in math. Also, use this time to establish norms of participating in a discussion-oriented classroom.<br><br>There are many other tasks from Boaler's website Youcubed, that can be used to address the mathematics practices and content of 7th grade. |
| **Proportional Relationships Cluster**<br><br>*NC.7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems.*<br><br>**NC.7.RP.1** Compute unit rates associated with ratios of fractions to solve real-world and mathematical problems.<br><br>**NC.7.RP.2** Recognize and represent proportional relationships between quantities.<br>a. Understand that a proportion is a relationship of equality between ratios.<br>• Represent proportional relationships using tables and graphs.<br>• Recognize whether ratios are in a proportional | **6 weeks** | **NC.7.EE.2**<br>**NC.7.EE.3**<br>**NC.7.EE.4**<br>**NC.7.NS.2** | Starting with **Proportional Relationships** gives more students an access point and an opportunity to engage with 7th grade mathematics, since the mathematics of finding unit rates and comparing ratios is the mathematics of comparing fractions.<br><br>Engaging in the mathematics of comparing fractions (finding unit rates, comparing ratios, determining proportionality) and multiplying a whole number by a fraction (the calculation of new dimensions via a scale factor), can lay the foundation for operating with a larger variety of rational numbers that will occur in the following |

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| relationship using tables and graphs.<br>• Compare two different proportional relationships using tables, graphs, equations, and verbal descriptions.<br>b. Identify the unit rate (constant of proportionality) within two quantities in a proportional relationship using tables, graphs, equations, and verbal descriptions.<br>c. Create equations and graphs to represent proportional relationships.<br>d. Use a graphical representation of a proportional relationship in context to:<br>• Explain the meaning of any point (x, y).<br>• Explain the meaning of (0, 0) and why it is included.<br>• Understand that the y-coordinate of the ordered pair (1, y) corresponds to the unit rate and explain its meaning.<br><br>**NC.7.RP.3** Use scale factors and unit rates in proportional relationships to solve ratio and percent problems.<br><br>*NC.7.G Draw, construct, and describe geometrical figures and describe the relationships between them.*<br><br>**NC.7.G.1** Solve problems involving scale drawings of geometric figures by:<br>• Building an understanding that angle measures remain the same and side lengths are proportional.<br>• Using a scale factor to compute actual lengths and areas from a scale drawing.<br>• Creating a scale drawing. | | | cluster, **Reasoning with Rational Numbers**.<br><br>NC.7.G.1 aligns with NC.7.RP.3 scale drawing and scale factor (See rationale on Progressions on Ratio and Proportional Relationships Document)<br>It is important to recognize how proportional reasoning will extend to students' work in solving equations with rational coefficients. Thus, NC.7.RP.3 connects to future NC.7.EE.2&3.<br><br>Another connection is between NC.7.RP.2 and the later cluster *Reasoning About Equations and Inequalities*. As students use equations to represent and compare proportions, they will engage with equations like y/x=k and y=xk, where k is the constant of proportionality. These equations can be reached by generalizing the proportional ratios within a ratio table. |
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| **Reasoning with Rational Numbers Cluster**<br><br>*NC.7.NS Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*<br><br>**NC.7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers, using the properties of operations, and describing real-world contexts using sums and differences.<br><br>**NC.7.NS.2** Apply and extend previous understandings of multiplication and division.<br>A. Understand that a rational number is any number that can be written as a quotient of integers with a non-zero divisor.<br>B. Apply properties of operations as strategies, including the standard algorithms, to multiply and divide rational numbers and describe the product and quotient in real-world contexts.<br>C. Use division and previous understandings of fractions and decimals.<br>● Convert a fraction to a decimal using long division.<br>● Understand that the decimal form of a rational number terminates in 0s or eventually repeats.<br><br>**NC.7.NS.3** Solve real-world and mathematical problems involving numerical expressions with rational numbers using | **6 weeks** | **NC.7.EE.2**<br>**NC.7.EE.3**<br>**NC.7.RP.3**<br>**NC.7.G.1** | Though this is the recommended second cluster, and NC.7.EE.2 and NC.7.EE.3 do not appear prior, the connection to the content in that standard exists in NC.7.NS.3<br><br>The understanding of operations with rational numbers will extend to the understanding of equations with rational coefficients, and how to solve them. Thus, these 7.NS.3 connects to the upcoming EE-cluster.<br><br>Prior to 7th grade, students will have operated with fractions, decimals, and negative integers (the introduction to integers occurs in 6th grade).<br><br>Ideally, the instructional tools utilized in 4th – 6th grade could again be used as a starting point for 7th graders as they increase their expertise with any rational number-type and begin to understand and utilize arithmetic algorithms. |

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| the four operations. | | | |
| **Probabilistic Reasoning Cluster**<br><br>*NC.7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems.*<br><br>*NC.7.SP Investigate chance processes and develop, use, and evaluate probability models.*<br><br>**NC.7.SP.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.<br><br>**NC.7.SP.6** Collect data to calculate the experimental probability of a chance event, observing its long-run relative frequency. Use this experimental probability to predict the approximate relative frequency.<br><br>**NC.7.SP.7** Develop a probability model and use it to find probabilities of simple events.<br>A. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.<br>B. Develop a probability model (which may not be uniform) by repeatedly performing a chance process and observing frequencies in the data generated.<br>C. Compare theoretical and experimental probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. | **3 weeks** | **NC.7.RP.1**<br>**NC.7.RP.2**<br>**NC.7.RP.3** | This cluster includes an application of proportional reasoning to the use of an experimental probability to "predict the approximate relative frequency", that is if the experimental probability of an event is ½, then the event is likely to occur in 10 out of the 20 trials.<br><br>Hence the **Proportional Relationships** cluster will need to precede this cluster.<br><br>See Progression on Probability and Statistics Document for order.<br><br>See Coherence Map for Probability and Statistics |

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| **NC.7.SP.8** Determine probabilities of compound events using organized lists, tables, tree diagrams, and simulation.<br>A. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.<br>B. For an event described in everyday language, identify the outcomes in the sample space which compose the event, when the sample space is represented using organized lists, tables, and tree diagrams.<br>C. Design and use a simulation to generate frequencies for compound events. | | | |
| **Reasoning About Expressions Cluster**<br><br>*NC.7.EE Use properties of operations to generate equivalent expressions.*<br><br>**NC.7.EE.1** Apply properties of operations as strategies to:<br>• Add, subtract, and expand linear expressions with rational coefficients.<br>• Factor linear expression with an integer GCF.<br><br>**NC.7.EE.2** Understand that equivalent expressions can reveal real-world and mathematical relationships. Interpret the meaning of the parts of each expression in context. | **3 weeks** | **NC.7.NS.1**<br>**NC.7.NS.2** | |

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| **Reasoning About Equations and Inequalities Cluster**<br><br>*NC.7.EE Solve real-world and mathematical problems using numerical and algebraic expressions, equations, and inequalities.*<br><br>**NC.7.EE.3** Solve multi-step real-world and mathematical problems posed with rational numbers in algebraic expressions.<br>• Apply properties of operations to calculate with positive and negative numbers in any form.<br>• Convert between different forms of a number and equivalent forms of the expression as appropriate.<br><br>**NC.7.EE.4** Use variables to represent quantities to solve real-world or mathematical problems.<br>A. Construct equations to solve problems by reasoning about the quantities.<br>• Fluently solve multistep equations with the variable on one side, including those generated by word problems.<br>• Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.<br>• Interpret the solution in context.<br>B. Construct inequalities to solve problems by reasoning about the quantities.<br>• Fluently solve multi-step inequalities with the variable on one side, including those generated by | **6 weeks** | **NC.7.NS.1**<br>**NC.7.NS.2** | The NC.7.NS.1 and NC.7.NS.2 standards should precede NC.7.EE.3, in the order of the clusters, so that students will be able to operate with rational coefficients.<br><br>Note that geometry is providing one context for the algebraic work of NC.7.EE, but the completion of NC.7.EE.4.b will require providing different contexts to complete the work of understanding inequalities and their solutions.<br><br>For more on the inequality relationship in NC.7.G.2 and the connection with algebra in NC.7.G.5 see Geometry, 7-8, High School Progression Document. |

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| word problems.<br>• Compare an algebraic solution process for equations and an algebraic solution process for inequalities.<br>• Graph the solution set of the inequality and interpret in context.<br><br>*NC.7.G Draw, construct, and describe geometrical figures and describe the relationships between them.*<br><br>**NC.7.G.2** Understand the characteristics of angles and side lengths that create a unique triangle, more than one triangle or no triangle. Build triangles from three measures of angles and/or sides.<br><br>*NC.7.G Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.*<br><br>**NC.7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve equations for an unknown angle in a figure. | | | |
| **Geometric and Measurement Reasoning Cluster**<br><br>*NC.7.G Solve real-world and mathematical problems involving angle measure, area, surface area, and volume*<br><br>**NC.7.G.4** Understand area and circumference of a circle.<br>• Understand the relationships between the radius, | **3 weeks** | **NC.7.NS.2**<br>**NC.7.EE.2**<br>**NC.7.RP.2** | This NC.7.G.4 standard denotes students' only explicit experience with circles in the 7th grade, within any content standards. The relationship between circumference and diameter of multiple circles connects to NC.7.RP.2. As students apply area and perimeter formulas, there is opportunity to connect to work with algebraic expressions, as well as operations with rational numbers |

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| diameter, circumference, and area.<br>• Apply the formulas for area and circumference of a circle to solve problems.<br><br>**NC.7.G.6** Solve real-world and mathematical problems involving:<br>• Area and perimeter of two-dimensional objects composed of triangles, quadrilaterals, and polygons.<br>• Volume and surface area of pyramids, prisms, or three-dimensional objects composed of cubes, pyramids, and right prisms. | | | when approximations of pi are utilized to calculate. |
| **Reasoning about Population Samples Cluster**<br><br>**NC.7.SP** Use random sampling to draw inferences about a population.<br><br>**NC.7.SP.1** Understand that statistics can be used to gain information about a population by:<br>• Recognizing that generalizations about a population from a sample are valid only if the sample is representative of that population.<br>• Using random sampling to produce representative samples to support valid inferences.<br><br>**NC.7.SP.2** Generate multiple random samples (or simulated samples) of the same size to gauge the variation in estimates or predictions, and use this data to draw inferences about a | **4 weeks** | **NC.7.RP.1**<br>**NC.7.RP.2**<br>**NC.7.RP.3** | The NC.7.RP cluster will need to precede this cluster, as it includes an application of proportional thinking: estimate a frequency or number in a population using a proportional relationship with a sample. (See rationale on Progressions on Ratio and Proportional Relationships Document)<br><br>The NC.7.SP.5-8 cluster should also precede this cluster, in particular NC.7.SP.5 may be used to connect to NC.7.SP.1 |

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| population with an unknown characteristic of interest. | | | |
| **Comparing Populations Cluster**<br><br>*NC.7.SP Make informal inferences to compare two populations.*<br><br>**NC.7.SP.3** Recognize the role of variability when comparing two populations.<br>A. Calculate the measure of variability of a data set and understand that it describes how the values of the data set vary with a single number.<br>● Understand the mean absolute deviation of a data set is a measure of variability that describes the average distance that points within a data set are from the mean of the data set.<br>● Understand that the range describes the spread of the entire data set.<br>● Understand that the interquartile range describes the spread of the middle 50% of the data.<br>B. Informally assess the difference between two data sets by examining the overlap and separation between the graphical representations of two data sets.<br><br>**NC.7.SP.4** Use measures of center and measures of variability for numerical data from random samples to draw comparative inferences about two populations. | **3 weeks** | | These NC.7.SP standards are distinct from NC.7.SP.1 and NC.7.SP.2 in part because of the focus on populations vs. samples. Instead of using information from a sample to make generalizations about a population, these standards require the calculation of population statistics to use to compare populations. |