Middle School Mathematics
Collaborative Instructional Framework

The following Collaborative Instructional Framework is meant to serve as a guide for teachers and districts as they organize the curriculum for the school year. Unlike traditional pacing guides, the instructional framework consists of clusters of standards that are meant to be adapted to various schools and contexts. The instructional framework used research on students' learning progression in mathematics to create and order clusters of standards that are taught together. While there is a strongly suggested order for teaching the clusters, we recognize that schools differ in their contexts and may wish to switch the order around. In those cases, we have given guidance regarding alternative clusterings; however, we note when certain clusters need to be taught in a certain order.

The Collaborative Instructional Framework was created over a five-month period, beginning in July. Twenty individuals from NC DPI, classroom teachers, district leaders, and university faculty worked together to a) read research about pacing guides, student learning progressions, and standards, b) determine the best clusterings per grade level based upon research, when possible, and c) wrote this draft of the framework. The members of this Middle School Framework Team include: Jen Arberg, Lisa Ashe, Stefanie Buckner, Caroline Butler, Chris Cline, Tara Costenoble, Dr. Deborah Crocker, Jill Hooley, Robert Leichner, Kim McCuiston, Dr. Katherine Mawhinney, Dr. Gemma Mojica, Nicolette Morgan, Joseph Reaper, Claudette Reep, Dr. Luke Reinke, Melanie Richey, Audrea Saunders, Patricia Shumaker, and Stacy Wozny. These mathematics professionals represent the four main regions of NC as well as urban, rural, and charter schools. Special thanks to Joseph Reaper and Lisa Ashe from NC DPI for providing guidance and checking for consistency among the framework and DPI resource documents.

CCSS.MATH.PRACTICE.MP1
Make sense of problems and persevere in solving them.

CCSS.MATH.PRACTICE.MP2
Reason abstractly and quantitatively.

CCSS.MATH.PRACTICE.MP3
Construct viable arguments and critique the reasoning of others.

CCSS.MATH.PRACTICE.MP4
Model with mathematics.

CCSS.MATH.PRACTICE.MP5
Use appropriate tools strategically.

CCSS.MATH.PRACTICE.MP6
Attend to precision.

CCSS.MATH.PRACTICE.MP7
Look for and make use of structure.

CCSS.MATH.PRACTICE.MP8
Look for and express regularity in repeated reasoning.

Standards for Mathematical Practice

The Standards for Mathematical Practice are critical ways of acting and communicating in classrooms that should be instilled in students throughout the school year. Whether students are learning to reason proportionally or statistically, they should be obliged to make sense of the problems posed (MP1) and create a mathematical solution that can contribute to their peers' and their own learning. When solving a problem, such as which company is the cheapest when comparing the prices of t-shirts, students should be able to create a viable argument for their choice, with mathematical evidence to defend their solution (MP3). Students should be able to move among various representations, reasoning quantitatively with symbols (MP2) and creat models of both everyday and mathematical situations they encounter (MP4). Teachers should provide opportunities for students to reason with a variety of tools (MP5), including technologies that are specific to mathematics (e.g., calculators, Desmos, GeoGebra, etc.). Attending to precision (MP6) is a practice in which students attempt to present clear arguments, definitions, and meanings for symbols as they explain their reasoning to others. Finding patterns and structure is crucial throughout the standards as students attempt to mathematize complex problem situations (MP7). Finally, students should attempt to find regularity in reasoning, such as recognizing that the slope is the coefficient of the x term in a linear equation.

8th Grade Mathematics Cluster

The clusters are recommended using the progression in the framework, but this is not the only possible progression teachers may use. Please look to the "Connections & Rationale" for notes about when one cluster must follow another, if another progression is desired. Also, continue to focus on how the Standards for Mathematical Practice can be incorporated with these content clusters. If considering changing the order of the clusters, we suggest keeping most of the sequence with the following two alternative orderings:

Recommended Order	Alternative A	Alternative B
Reasoning about Similarity and Transformations Functional Reasoning/System unit Reasoning about Equations/Angles unit Statistical Reasoning unit Reasoning with Real Numbers/ Pythagoras/Volume unit Reasoning with Exponents/Scientific Notation unit	Reasoning about Similarity and Transformations *Functional Reasoning/System unit* *Statistical Reasoning unit* *Reasoning about Equations/Angles unit* Reasoning with Real Numbers/ Pythagoras/Volume unit** Reasoning with Exponents/Scientific Notation unit** ** Real numbers and exponents units can be switched according to your preference	Reasoning about Similarity and Transformations *Reasoning about Equations/Angles unit* *Functional Reasoning/System unit* *Statistical Reasoning unit* Reasoning with Real Numbers/ Pythagoras/Volume unit** Reasoning with Exponents/Scientific Notation unit** ** Real numbers and exponents units can be switched according to your preference

Please pay attention to the "Supporting Standards" and "Connections & Rationale" portions of this recommendation. These tools should help to connect mathematical concepts across the units, and to highlight potential opportunities for revisiting previous units' main ideas.

Standards/Cluster	Recommended Timeframe	Supporting Standards	Important Notes
Jo Boaler's Week of Inspirational Math Week 3 **-Or-** **Other problem solving and environment-building activities**	**1 week**		The intention of the first week(s) of class is to establish a mindset that math is about patterns and struggle is good in math. Also, use this time to establish norms of participating in a discussion-oriented classroom. There are many other tasks from Boaler's website Youcubed, that can be used to address the mathematics practices and content of 8th grade.
Reasoning about Similarity and Transformations Cluster **(Start with transformations, then define congruence/similarity)** **NC.8.G.3** Describe the effect of dilations about the origin, translations, rotations about the origin in 90 degree increments, and reflections across the x-axis and y-axis on two-dimensional figures using coordinates. **NC.8.G.2** Use transformations to define congruence. • Verify experimentally the properties of rotations, reflections, and translations that create congruent figures. • Understand that a two-dimensional figure is congruent to another if the second can be obtained	**4 weeks**	**7th grade standards on similarity and ratio** **7.G.1** **8.G.5**	The 8th Grade course begins with transformations and similarity as it relates to the 7th grade standards involving similarity and proportional relationships. It is accessible to all students and can involve many hands-on, visual activities. Several common, aligned curricula have made the decision to start with these standards. Prerequisite Skills: Graphing Points (6.NS.8, 6.G.3), Solving Proportions (7.RP.2), Understanding Angle Measure (7.G.5), Understanding Scale Drawing (7.G.1) Connections Within Course: This comes before functions because the idea of similar triangles will aid in the concept of slope (with slope being the

from the first by a sequence of rotations, reflections, and translations. • Given two congruent figures, describe a sequence that exhibits the congruence between them. **NC.8.G.4** Use transformations to define similarity. • Verify experimentally the properties of dilations that create similar figures. • Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. • Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.			same given any two points on the line). Additionally, 8.G.5 is a supporting standard, as the angle relationships should be established through the use of transformations. In the next unit, the standard will be covered in depth and applied in the context of solving equations.
Functional Reasoning Cluster **NC.8.F.1** Understand that a function is a rule that assigns to each input exactly one output. • Recognize functions when graphed as the set of ordered pairs consisting of an input and exactly one corresponding output. • Recognize functions given a table of values or a set of ordered pairs. **NC.8.F.2** Compare properties of two linear functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	**8 weeks**	**Builds on grade 7 equations**	Functions are taught separate from equations to ensure that students understand the concept and graphical representations of functions independently of the algebraic solutions to equations. Systems (8.EE.8) are included as well, as the revised standards only require solving by graphing. The understanding of the intersection on the graph being the point where the input and output of functions are equal provides a conceptual link between the two strands. Prerequisite Standards: Understanding Proportional Graphs (7.RP.2), Solve Multi-Step Equations (7.EE.4), Evaluate Expressions (6.EE.2),

NC.8.F.3 Identify linear functions from tables, equations, and graphs. **NC.8.F.4** Analyze functions that model linear relationships. • Understand that a linear relationship can be generalized by $y = mx + b$. • Write an equation in slope-intercept form to model a linear relationship by determining the rate of change and the initial value, given at least two (x, y) values or a graph. • Construct a graph of a linear relationship given an equation in slope-intercept form. • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of the slope and y-intercept of its graph or a table of values. **NC.8.F.5** Qualitatively analyze the functional relationship between two quantities. • Analyze a graph determining where the function is increasing or decreasing; linear or non-linear. • Sketch a graph that exhibits the qualitative features of a real-world function. **NC.8.EE.8** Analyze and solve a system of two linear equations in two variables in slope-intercept form. • Understand that solutions to a system of two linear equations correspond to the points of intersection of their graphs because the point of intersection satisfies both equations simultaneously.			Representing Ratio Problems (6.RP.3) and Relationships between independent and dependent variables (6.EE.9) Connections Within Course: Functions is recommended here to follow directly after the transformations to continue building on the work with graphical relationships and set up the linear equations to follow. However, the functions/systems cluster can also be taught after equations to put context and graphical representation to the equations. Either way, functions should be taught before statistics (either 1 or 2 units before) to set up the graphical representation of scatterplots and best-fit lines.

• Solve real-world and mathematical problems leading to systems of linear equations by graphing the equations. Solve simple cases by inspection.			
Reasoning about Equations and Angles Cluster **(Integrate solving equations with geometric angle relationships.)** **NC.8.EE.7** Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable. • Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions. • Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides. **NC.8.G.5** Use informal arguments to analyze angle relationships. • Recognize relationships between interior and exterior angles of a triangle. • Recognize the relationships between the angles created when parallel lines are cut by a transversal. • Recognize the angle-angle criterion for similarity of triangles. • Solve real-world and mathematical problems involving angles.	**6 weeks**	**Supporting standards in transformations** **8.G.4**	Equations and angles are taught together to provide context for both - solving for related angle measures provides a context for some equations. Prerequisite standards: Angle relationships (explored in the Transformations cluster, 8.G.5), Setting Up and Solving Multi-Step Equations and Inequalities (7.EE.4), Understanding of Angle Relationships in Triangles (7.G.2) Connections Within Course: The Equations/Angles cluster is presented here to build on the graphical representations presented in the Transformations and Functions clusters by applying the algebraic solutions. It can also be taught before Functions, as noted in the Functions cluster. This cluster also emphasizes the importance of presenting the angle relationships in the Transformations/Similarity cluster. As students discover these relationships (AA, parallel lines) through transformations and similarity, they will naturally be able to apply the relationships to solve equations for the angle measures.

			It will be essential to use other contexts as well in solving these equations, but the angles provide a natural link between the geometry and equations standards to further integrate the course and ensure conceptual understanding.
Statistical Reasoning Cluster **(Please remember to incorporate Function standards as a support/re-loop in the statistics standards.)** **NC.8.SP.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Investigate and describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **NC.8.SP.2** Model the relationship between bivariate quantitative data to: • Informally fit a straight line for a scatter plot that suggests a linear association. • Informally assess the model fit by judging the closeness of the data points to the line. **NC.8.SP.3** Use the equation of a linear model to solve problems in the context of bivariate quantitative data, interpreting the slope and *y*-intercept. **NC.8.SP.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.	**4 weeks**	**NC.8.EE.7**	This unit should follow the functions/systems cluster, whether it is taught before or after equations. Any equations that will be solved in this unit will be multi-step equations learned in 7th grade, so it is not necessary to teach equations before statistics. Patterns within a group of data, such as association, are presented as important concepts in analyzing statistical data. Prerequisite Standards: Setting Up and Solving Multi-Step Equations (7.EE.4), Understand Percents for Relative Frequency (6.RP.4), Analyzing Statistical Samples (7.RP.2), Relationships between dependent and independent variables (6.EE.9) Connections Within Course: The statistics presented in 8th grade connect directly to the functions work, as students will generalize a linear function from given bivariate data and analyze the function based on its slope, y-intercept, and association. The statistics unit will also help

• Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. • Use relative frequencies calculated for rows or columns to describe possible association between the two variables.			students connect functions to real-world situations.
Reasoning with Rational/Irrational numbers (and Pythagorean Theorem) Cluster **NC.8.NS.1** Understand that every number has a decimal expansion. Building upon the definition of a rational number, know that an irrational number is defined as a non-repeating, non-terminating decimal. **NC.8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers and locate them approximately on a number line. Estimate the value of expressions involving: • Square roots and cube roots to the tenths. • π to the hundredths. **NC.8.EE.2** Use square root and cube root symbols to: • Represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number.	**8 weeks**	**NC.8.EE.7**	This unit incorporates NS, EE, and G standards. They might seem different, but the mathematical concepts in each standard relate to each other and provide context for students. Traditionally, many courses have taught laws of exponents and scientific notation prior to the concepts in this unit. That can work, but please ensure that the Pythagorean Theorem, square roots, and volume formulas are understood conceptually to help their future understanding in high school math, rather than just as applications of exponents. Prerequisite Standards: Evaluate expressions with exponents (6.EE.1), Find distances on the coordinate plane between points with the same x or y coordinate (6.NS.8), Volume of right prisms and pyramids (7.G.6), Circles (7.G.4). Connections with course: The values of rational and irrational numbers, when developed through areas of squares, will conceptualize squares and

• Evaluate square roots of perfect squares and cube roots of perfect cubes for positive numbers less than or equal to 400. **NC.8.G.6** Explain the Pythagorean Theorem and its converse. **NC.8.G.7** Apply the Pythagorean Theorem and its converse to solve real-world and mathematical problems. **NC.8.G.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **NC.8.G.9** Understand how the formulas for the volumes of cones, cylinders, and spheres are related and use the relationship to solve real-world and mathematical problems.			square roots for students in their first exposure to the concept. Solving equations using square and cube roots gives context to the NS standards, and the volume formulas give further context to the EE standards. Furthermore, the Pythagorean Theorem is a key mathematical idea that connects rational vs. irrational numbers, estimating square roots, solving equations with square roots, and distance and graphing. It can connect back to the graphing work from the first cluster and bring together a lot of the major work of the grade when presented late in the course.

Reasoning about Exponents/Scientific Notation Cluster **NC.8.EE.1** Develop and apply the properties of integer exponents to generate equivalent numerical expressions. **NC.8.EE.3** Use numbers expressed in scientific notation to estimate very large or very small quantities and to express how many times as much one is than the other. **NC.8.EE.4** Perform multiplication and division with numbers expressed in scientific notation to solve real-world problems, including problems where both decimal and scientific notation are used.	**3 weeks**		These three standards are fairly isolated from the major work of 8th grade, and they can be taught before the previous cluster (rational/irrational numbers and Pythagorean Theorem). This cluster should be based on the concept of exponents that students have previously built. The exponent laws are foundational for future courses as students study rational exponents, quadratics, and polynomials. Prerequisite Standards: Evaluate expressions with exponents (6.EE.1) Connections Within Course: A basic understanding of exponents has been built in 6th grade, expanded in 7th grade, and applied in 8th grade through the Pythagorean Theorem and volume formulas, so the main connections of this specific unit relate to future courses. Exponent rules are not necessary to understanding the concepts involved in solving equations using square and cube roots or the Pythagorean Theorem, but they will enhance the understanding of the application and concept of exponents when taught in successive units.