NC Math 1 – Similarity & Congruence

NC Collaborative for Mathematics Learning

Research-Practice Brief #8

SIMILARITY AND CONGRUENCE

When students reason with geometry, they utilize *given geometric characteristics* to explain relationships within and between geometric objects. The objects of focus in this unit of the Collaborative Pacing Guide for NC Math 2 include but are not limited to triangles and lines, building on the exploration of triangles and parallel lines that students engage with in 8th grade The geometry of NC Math 1 is focused on orienting figures in the plane and using coordinates to calculate geometric characteristics.

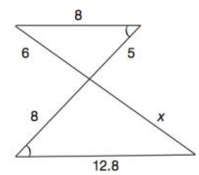
Students in NC Math 2 will understand *similarity* and *congruence* as characteristics of pre-images and images of transformations. Connecting the standards NC.M2.G-SRT.1-3 with the *Function* domain, NC Math 2 students will experiment with dilations to verify the resulting similarity of the output of a figure that has been input into a non-rigid transformation. (See the NC Math 2 Unit 1 brief for more on transformations as functions.) Rigid transformations (translations, rotations, and reflections) will be utilized as students engage in learning opportunities around NC.M2.G-CO.6-8, as students justify congruency and connect to the criteria for triangle congruency (ASA, SAS, SSS, HL).

NC.M2.G-CO.9,10 call for students to ***prove*** theorems about lines, angles, and triangles, as well as use these relationships to prove other claims about geometric figures. The HS Instructional Framework provides examples of theorems addressing this content.

A TRANSFORMATION PERSPECTIVE

How would you define ***similarity?*** Students often encounter a ***static*** perspective, where similarity is seen as a discrete numeric relationship between two figures (e.g. corresponding angle measures are equal or corresponding side lengths are proportional). Alternatively, one could take a geometric ***transformation*** perspective and focus on enlarging or reducing figures proportionally and conceptualize similarity as created by performing a sequence of transformations on the other.

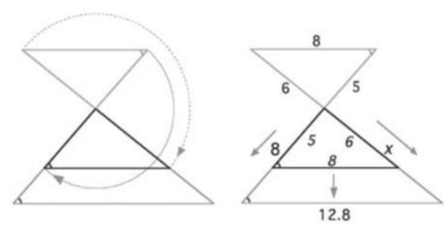
Research has shown that students who do not have a strong understanding of transformations perform poorly on similarity tasks (Seago et al., 2013). Consider the following problem taken from a previous NAEP exam.



9. In the figure above, the two triangles are similar. What is the value of x?

A static perspetive of similarity would require that students determine which sides of the triangle are corresponding, which may be challenging to students who access mathematics from a spatial, geometric perspective.

However, one could also approach the problem from a transformation perspective, visualizing the upper triangle rotating 180° around the common vertex, recognize the sides that are collinear, and draw upon their knowledge of dilation to solve both this problem and all possible similar triangles in this case.



THE IMPORTANCE OF PROOF & STUDENT REASONING

A ***mathematical proof*** is a coherent argument built of mathematical facts and relationships that support a conclusion. When students are asked to ***prove*** theorems in mathematics, the expected product is often more sophisticated than what students create when asked to ***verify***, ***demonstrate, apply***, ***explain***, or ***justify***. Thus, it is important to clearly communicate such expectations with students if we intend for a ***proof*** to be distinct from an ***answer*** or a ***solution***.

All too often, proof is treated as separate from the rest of mathematics; however, proof is central to the work of mathematicians and should be an essential component of mathematics instruction. A logical next step to verifications or justifications would be a formal proof that illustrates that a conjecture will always hold true. This can be done in many forms, for example, the traditional two-column, flow diagram, and/or paragraph proofs. In the context of similarity and congruence, a proof could be identifying a finite sequence of transformations that map the pre-image onto the image.

THE IMPORTANCE OF MAKING PROOF VISIBLE

Proofs are often presented as finished products and in doing so students miss out on experiencing the process of creating the proof. It is important to provide students opportunities to explore and make conjectures prior to formal proof, as this is often where important insights come about that ultimately become the building blocks of a formal proof (Raman, 2003).

Research has shown students often view proofs as a proof of only the single case of the problem they are proving and thus do not hold strong conceptions that "guarantee safety from counterexamples" (Chazan, 1993, p. 382). Students need time experimenting to make these conjectures. Through the use of dynamic software (e.g. geogebra.com, desmos.com/geometry), students have opportunities to construct infinite examples; moving them away from the notion that a relationship holds for just one case.

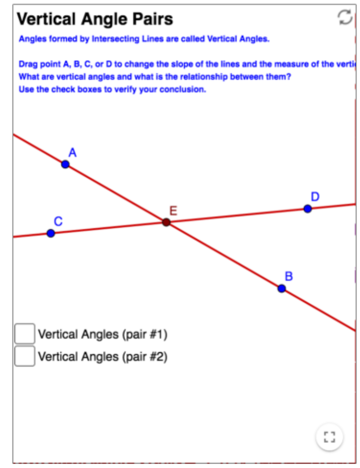
To support students, Driscoll et al. (2007) offers a geometric habits of mind framework to use as a guide for the kinds of tasks and questioning teachers can use in order to develop students' geometric thinking. Instruction should provide students with opportunities to:

- look for relationships within and between figures;
- generalize geometric ideas;
- investigate invariants (things that stay the same); and
- provide equal opportunities for both exploration and reflection.

Students will need to be provided tools for investigating geometric relationships, keeping in mind that different tools may better highlight different paths to justification. In addition to technology tools, you may consider using tracing paper, compass, straightedge, or mirrors.

AN EXAMPLE: THE VERTICAL ANGLE THEOREM

The vertical angle theorem is identified as important in this unit. Using a preconstructed GeoGebra file, students can explore the relationships that exist with vertical angles prior to formal proof. For example, the [Exploring Vertical Angles task] allows students to drag points on two intersecting lines to change the angles they form. In addition, they can turn different pairs of vertical angles. Asking students about what they notice and wonder prior to showing the angle pairs and their measures and then again after would support their conjecture building and identification of key ideas to a formal proof.



By exploring the linear pairs of angles and vertical pairs students might notice that the vertical angle pairs are made up of one angle each from two linear pairs. This provides an important insight to a formal proof of the congruence of vertical angles. If your students have not had much experience with constructing formal proofs at this point, you might consider using partial proofs (fill in the blanks) or jig saw proofs as a first step to constructing their own logical arguments.

QUESTIONS TO CONSIDER WITH COLLEAGUES

- *How does a students' understanding of similarity from a transformational perspective support their understanding of proof?*
- *How can you support your students in developing stronger geometric habits of mind?*
- *What questions would you ask a student who believes to have proved something by providing a single example?*
- *Thinking about the similar triangle example discussed on the front page, how does this approach support a student's understanding of both similarity and proof?*

REFERENCES

Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. *Educational Studies in Mathematics*, *24*(4), 359-387.

Driscoll, M. J., DiMatteo, R. W., Nikula, J., & Egan, M. (2007). *Fostering geometric thinking: A guide for teachers, grades 5-10*. Portsmouth, NH: Heinemann.

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