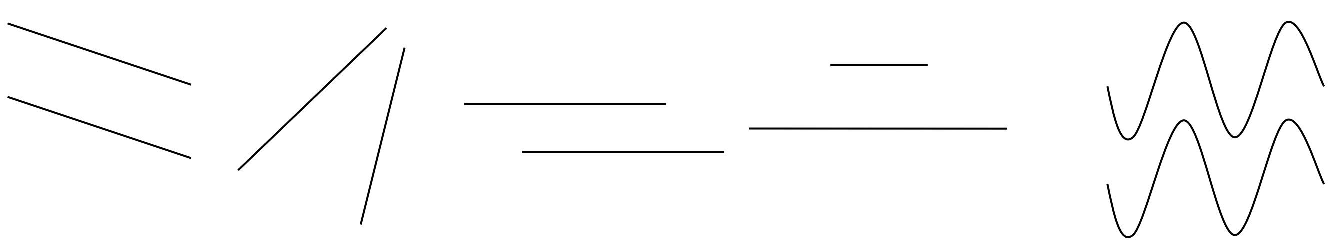
Angle Tasks 1: Parallel Lines and Transversals

Framework Cluster	**Reasoning About Equations and Angles**
Standard(s)	**8.G.5 Use informal arguments to analyze angle relationships** • ~~**Recognize the relationships between interior and exterior angles of a triangle**~~ • **Recognize the relationships between the angles created when parallel lines are cut by a transversal** • ~~**Recognize the angle-angle criterion for similarity of triangles**~~ • ~~**Solve real-world and mathematical problems involving angles**~~
Materials/Link	**Colored Pencils, Patty Paper, Protractors, Copy of Task Student Pages Below**
Learning Goal	- **Students will discover relationships, by modeling, between angles created when parallel lines are cut by a transversal.** - **Students will apply angle relationships to determine measures of related angles when parallel lines are cut by a transversal.**

Task Overview: In this task, students will discover relationships between corresponding angles, alternate exterior angles, and alternate interior angles using patty paper or tracing paper. They will then apply what they learned in 7th grade about vertical angles and in Unit 1 about transformations to extend these relationships to alternate interior and alternate exterior angles. Finally, after discovering the relationships, they will apply them to find angle measures when one is given. Throughout the task, students create their own definitions for the related pairs based on what they see and discover the relationships, which will help them remember and apply them as they move through the unit and begin solving equations for the related angles.

Prior to Lesson: This task is an introduction to the 8th grade angle relationships, and the questioning strands require no previous 8th grade knowledge of angles. Knowledge of transformations from earlier in 8th grade, and knowledge of vertical angles and linear pairs from 7th grade, are important, but this could be the “Day 1” of the angles unit.

<u>Teaching Notes</u>
Task Launch: You might pose a variety of different pictures and ask the class to determine if the lines are parallel or not. Show them one at a time as a way to help students clarify their definition of parallel.



Then, introduce the first page of the activity sheet below. Students should work in partners or small groups. Follow up with a whole class discussion.

Smith and Stein's 5 Practices for Implementing Math Tasks are appropriate for assessing student work and setting up a whole-class discussion.

The final page of the task can be used as a check for understanding to see if students are ready to move on to equations for the angles or if they need more help with the relationships, or it can be used as part of the discovery with a similar page used as an exit ticket.

Anticipate Student Reasoning:

In the launch activity, most students will claim the first set is parallel because they never cross. In the second set, most students will say no because they WILL cross at some point. In the third set, some students may argue they are not parallel because they do not have the same starting and ending point. This may be an opportunity to capitalize on students who argue about the space between the lines being the same distance apart. Or talk about parallel line segments versus parallel lines (line segments are parallel if a set of parallel LINES can be traced across them). In the fourth set, may also provoke students' doubt because sometimes students think that parallel lines have to be the same length. Again, the discussion can center on parallel LINES versus segments. Finally, the last set may spark students to look at distance apart rather than never crossing. (The last set are parallel curves).

For the activity sheet, students could use lots of strategies to discover the relationships. Their answers on the first two pages of the task will vary, but correct answers will ultimately determine that corresponding, alternate interior, and alternate exterior angles are congruent when lines are parallel. Students can use protractors, patty paper, or other modeling strategies to see these relationships. Look for students who use rigid motions or describe using rigid motions in their justifications.

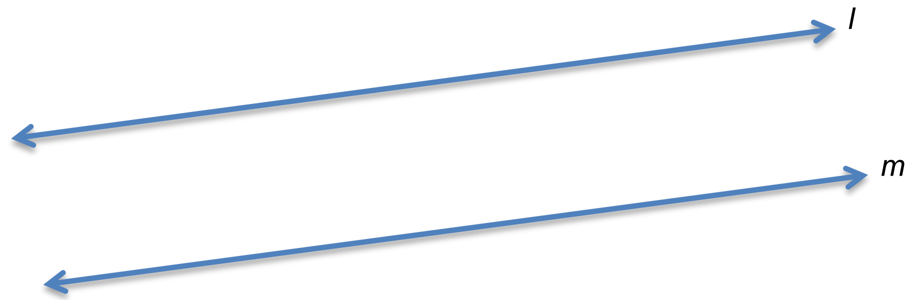
Having student clarify the terms *corresponding, alternate interior and alternate exterior* may be helpful, especially for EL students.

Challenge students on 3b to find a counter example (e.g., non-parallel lines).

Student task sheets begin on next page.
Adapted from pg. 18-19 of Utah Middle School Math Project:
https://eq.uen.org/emedia/file/04e4c5cf-8993-47e1-8fbfab041db84974/1/8Ch10Student_Workbook.pdf

Parallel Lines and Transversals

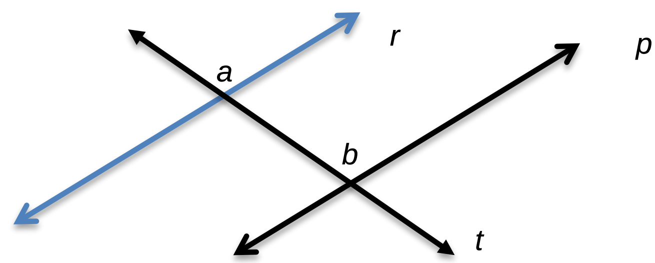
1. Use the picture given below to describe what parallel lines are. Use the correct notation to denote that the line *l* is parallel to line *m.*



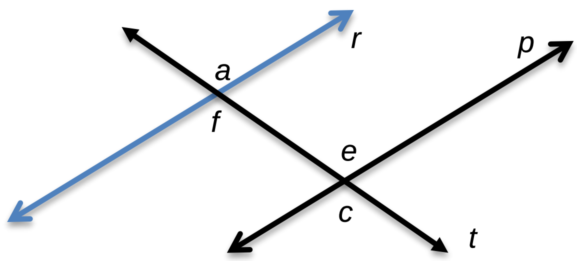
2. Draw a line across the two parallel lines above and label it line *t*. Line t is called a transversal because it transverses, or crosses, the other two lines. How many angles are created by line *t*?

3. Use what you know about rigid motions to answer the following questions about the parallel lines *p* and *r* cut by transversal *t.*

   a. $\angle a$ and $\angle b$ are called *corresponding angles*. Do you think corresponding angles are congruent or not congruent? Use tracing paper and your knowledge of rigid motions to determine your answer.



   b. Do you think ALL corresponding angles behave this way? Explain.

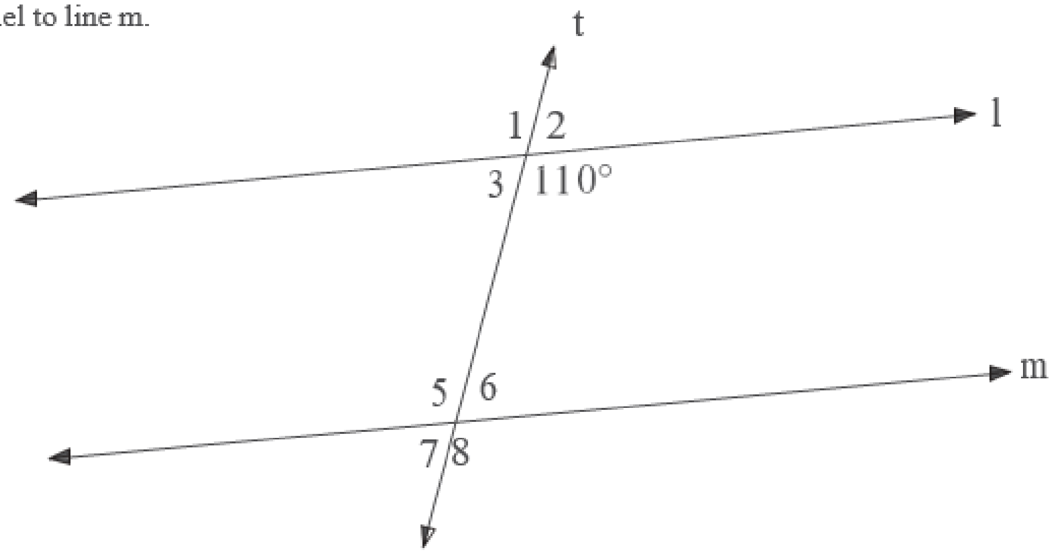


1. $\angle c$ and $\angle a$ are called *alternate exterior angles*. Do you think alternate exterior angles are congruent or not congruent? Use tracing paper and your knowledge of rigid motions to determine your answer.

2. $\angle e$ and $\angle f$ are called *alternate interior angles*. Do you think alternate interior angles are congruent or not congruent? Use tracing paper and your knowledge of rigid motions to determine your answer

3. Did you notice $\angle c$ and $\angle e$? How are they related? Explain.

In the diagram below one angle measure is given. Find the measure of each remaining angle if line l is parallel to line m.



a. $m\angle 1 =$ ______

b. $m\angle 2 =$ _____

c. $m\angle 3 =$ ______

d. $m\angle 5 =$ ______

e. $m\angle 6 =$ ______

f. $m\angle 7 =$ ______

g. $m\angle 8 =$ ______

Angles Task 2: Angle-Angle (AA) Relationships	
Framework Cluster	**Reasoning About Equations and Angles**
Standard(s)	**8.G.5 Use informal arguments to analyze angle relationships** ● **Recognize the relationships between interior and exterior angles of a triangle** ● ~~**Recognize the relationships between the angles created when parallel lines are cut by a transversal**~~ ● **Recognize the angle-angle criterion for similarity of triangles** ● ~~**Solve real-world and mathematical problems involving angles**~~
Materials/Link	**Rulers, Protractors, Colored Pencils, Copy of Student Task Below**
Learning Goal	- **Students will discover that two triangles with two congruent corresponding angles are similar through a modeling activity.** - **Students will discover some similarities and differences between right triangles.**

Task Overview: Students will begin by drawing two triangles of different sizes with two angle measures given. They will measure the lengths of sides and the “other” angle in each triangle, and discover mathematically that the triangles are similar. They will then repeat the activity with right triangles, with the right angle given and one other angle measure.

Prior to Lesson: Students worked with similar figures on the coordinate plane in Unit 1 with dilations, and now they are formalizing the AA relationship for similarity. A review of angles, either by teaching Task 1 and its associated lessons first or a brief review of interior and exterior angles of triangles, could be appropriate, but this is a potential introductory task for the angle relationships in similar triangles.

Teaching Notes

Task Launch: Question #1 of the task is an appropriate launch, as it will allow the teacher to assess (and re-teach if necessary) the concepts of angle measure, how to use a protractor, and create appropriate groups for the activity. Additionally, the end of Question #1 is an important informal “check-in” for the students, because if the triangles aren’t drawn correctly, the students will not see the mathematical relationships later in the task.

Directions: Students can mostly be “set free” in partners or groups, depending on classroom setup and routines, to complete the remainder of the task. Important things for the teacher to monitor are accurate use of the mathematical tools, the answers specifically to Part 1 Question #4 (that students notice the similarity) and Part 2 Question #3 (that students notice the similarity), and that the student discourse in the groups incorporates all notice/wonder responses. Strategies such as assigning group roles or expecting every group member to give a 30 second response to each question will help make sure all voices and ideas are heard. Another possible formative assessment could be asking groups to draw their triangles on poster board/chart paper with the associated lengths and ratios, and students could conduct a gallery walk to see that no matter how large or small the similar triangles are, they share the relationships.

The last two questions can be the basis for a whole class discussion, as they comprise the main mathematical concepts in the task. Smith and Stein’s 5 Practices for Implementing Math Tasks are

appropriate for assessing student work and setting up a whole-class discussion, but small-group discussion should be expected first.
Possible Strategies/Anticipated Responses: Actual answers will vary because the sizes of the triangles and student thoughts will vary, but ultimately the answers should lead to the realization that triangles with two pairs of equal corresponding angles are similar and right triangles are NOT all similar, unless one pair of acute corresponding angles is also equal. Inaccurate measurement, both with rulers and protractors, could be an issue for students. Setting norms for the measurements (nearest millimeter, nearest degree, etc.) will help, but decimal approximations could still be off. While monitoring group work on the task or during the whole-class discussion, it will be important to point out reasons for these small differences, but encourage students that if they had been using a computer, they would get exact responses. (A follow-up activity, either projected or having students complete a similar activity through Geogebra, Desmos, or a similar app, could show them how the angles and ratios really are exactly equal.)

Student task sheets begin on next page.

Angles Task - Angle-Angle (AA) Relationships

Part I: Drawing Triangles

a) On separate paper, draw triangle ΔABC, with m<A = 30^0 and m<B = 50^0. The triangle can be any size.

b) Then, draw triangle ΔDEF, with m<D = 30^0 and m<E = 50^0. The triangle can be any size, but make it either bigger or smaller than the first one.

Find the following measures using a ruler and protractor:

AB = _____ BC = ______ AC = ______ m<C = ______

DE = _____ EF = ______ DF = ______ m<F = ______

1. What do you notice or wonder about the lengths and angle measures? What hypotheses can you make about the triangles?

2. Divide DE/AB, EF/BD, and DF/AC. What do you notice or wonder? What might that mean about the triangles?

Part II: Right Triangles

1. a) On separate paper, draw triangle ΔGHI, with m<G = 60^0 and m<H = 90^0. The triangle can be any size.

   b) Then, draw triangle ΔJKL, with m<J = 60^0 and m<K = 90^0. The triangle can be any size, but make it either bigger or smaller than the first one.

2. Find the following measures using a ruler and protractor:

GH = _____ HI = ______ GI = _______ m<I = ______

JK = _____ KL = ______ JL = _______ m<L = ______

3. What do you notice or wonder about the length and angle measures? What hypotheses can you make about the triangles?

4. Do you notice any similarities or differences from your Part I triangles?

Part III: Conclusions

1. Based on what you know about triangles, why are the measures of the third angles equal if the other two angle measures are equal?

2. Are all right triangles similar? Why or why not?

Angles Tasks 3: A Triangle's Interior Angle	
Framework Cluster	**Reasoning About Equations and Angles**
Standard(s)	**8.G.5 Use informal arguments to analyze angle relationships** ● **Recognize the relationships between interior and exterior angles of a triangle** ● **Recognize the relationships between the angles created when parallel lines are cut by a transversal** ~~● **Recognize the angle-angle criterion for similarity of triangles**~~ ~~● **Solve real-world and mathematical problems involving angles**~~
Materials/Link	**Student Printout of Task Below** **Original Illustrative Math Task Link (with further explanation and teaching strategies): https://www.illustrativemathematics.org/content-standards/8/G/A/5/tasks/1501**
Learning Goal	- **Students prove that the interior angles of a triangle add to 180 degrees using parallel line and transversal relationships.**

Task Overview: This task can be viewed as a cornerstone to the entire angle unit, as students will apply their learning about parallel lines and transversals, interior angles of triangles, and relationships of adjacent angles and lines to prove that the interior angles of a triangle add to 180 degrees. It is a very open-ended task, so student thought and discovery is encouraged.

Prior to Lesson: Students will need to understand relationships between angles formed by parallel lines and transversals and understand that adjacent angles on a straight line add to 180 degrees. This task could be used as part of an assessment of this group of lessons, as the thought required to combine the concepts is the key connection.

Teaching Notes (more ideas and commentary given in the Illustrative Mathematics link):
Task Launch: An appropriate launch for this task could ask students to list everything they know about angles and share with their groups, potentially creating a master list for the class going into the task. In this way, students will review all the pertinent information, but they will not be guided to the solution. (Note: A warm-up about parallel lines and transversals, alternate interior angles, or linear pairs would NOT be ideal, as it would guide student thinking to the solution to the task before they have the opportunity for productive struggle.

Directions: While it is important that students share their thoughts and come to an ultimate understanding in pairs or groups on this task, individual think/task time is necessary at the beginning to ensure that some students aren't stifled by the "groupthink" that sometimes occurs in groups. If students are given 10 minutes to work by themselves and annotate the problem/begin to write ideas, then switch papers with other students to discover other thinking, it will ensure that all students' ideas can be involved in the class understanding.

Ultimately, as groups begin to work together to determine a means of proving the relationship of the interior angles, they can be encouraged to present their proof format in any form (written, step-by-step using pictures, substituting numbers, etc.) Formal proof is not an expectation in 8th grade, and any clear way that the students can use to demonstrate their understanding is appropriate.

The final question could serve as the whole-class discussion at the end of the task. Especially for students who might have used numbers to help clarify their proof, this generalized the process to all triangles using the relationships from the proof itself. Students probably already know that all interior angles of a triangle add to 180 degrees, but this generalization helps with the “why” of the concept and will help reinforce the parallel line-transversal relationships.

Possible Strategies/Anticipated Responses: A written solution is presented on the Illustrative Mathematics link: https://www.illustrativemathematics.org/content-standards/8/G/A/5/tasks/1501

However, as stated in the directions, other mathematically correct presentations of the solution could be appropriate.

Some students could struggle identifying the correct alternate interior angles, because the sides of the triangle create two transversals. If this is seen as the teacher monitors the student work, it could be suggested that the students consider each transversal individually.

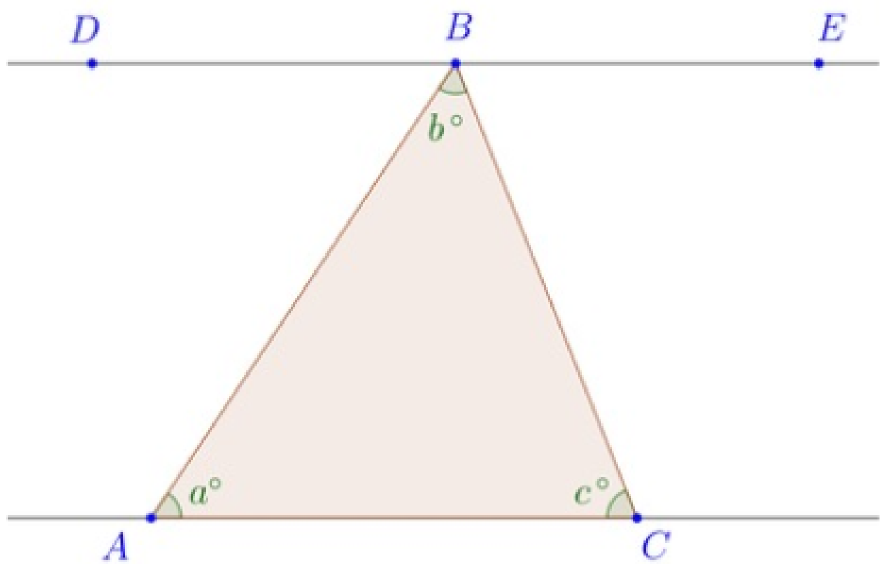
Also, for students that have trouble getting started because of the variable angle measures presented in the task, it could help to suggest they assign numbers to each of the angles. Hopefully, they will realize that the angles must add to 180 degrees (as the interior angles of a triangle), but if that is still an issue then that can be a second hint. Any hints past that should come from student discussions in their groups.

Student task sheets begin on next page.

8.G A Triangle's Interior Angles

Task

Given that $\overleftrightarrow{DE} \parallel \overleftrightarrow{AC}$ in the diagram below, prove that $a + b + c = 180$.



Explain why this result holds for any triangle, not just the one displayed above.