Fourth Grade Instructional Framework

Cluster 1	Build a math community through real data
Cluster 2	Explore multiplicative comparison, area and perimeter, factors, and multiples
Cluster 3	Use place value strategies to add and subtract whole numbers
Cluster 4	Develop multiplication/division strategies
Cluster 5	Extend the understanding of fractions
Cluster 6	Connect to decimal notation
Cluster 7	Understand operations with fractions and decimals
Cluster 8	Apply geometric concepts
Cluster 9	Use place value to understand metric measurement

Introduction

The purpose of this document is to connect and sequence mathematical ideas to enable teachers to plan learning opportunities for students to develop a coherent understanding of mathematics. **Clusters** and sequencing are designed to foster students' meaning making of the connections among mathematical ideas and procedures. This meaning making occurs over time. Therefore, the concepts are included in multiple clusters with increasing depth. They build across the year beginning with conceptual understanding and moving toward procedural fluency.

Each cluster includes a list of related **content standards** and a range of **suggested duration**. Standards indicate the mathematics expectations of students by the end of the school year. Standards are introduced and developed throughout the year, so the fact that a content standard is listed in a particular cluster does not indicate that it is to be mastered in the cluster. In some clusters, strikethroughs in the content standards denote the portion of the standard that will be taught later. In other clusters, the full standard appears, but suggestions about the intended focus are noted in the cluster descriptions. Because standards may be included in clusters long before mastery is expected, formative assessment is an essential tool for instructional planning and reporting student progress. This assessment naturally occurs as teachers elicit students' mathematical thinking and reasoning while doing mathematics.

Particular **Standards for Mathematical Practice** are indicated in bold for each cluster. The suggestions are a guide for teachers. While the bolded practices may lend themselves particularly well to the cluster's content, this does not imply that they are the only practices students will use. Students doing rich mathematical tasks will naturally engage in many mathematical practices as they do mathematics. During instruction teachers may observe and decide to highlight the other practices students are using beyond those bolded in the cluster.

Each cluster includes a section called **"What is the mathematics?"** that describes the significant concepts and connections within the standards necessary for students to make sense of and use the mathematics. A second section called **"Important Considerations"** provides guidance based on student learning progressions as well as ideas and models for teaching within problem-solving situations. Problem-solving and mathematical reasoning define what it means to do mathematics. Rich tasks (including word problems) provide students with concrete contexts to use as they are introduced to new mathematics. Later, work within such tasks allows students to develop understanding and eventually to demonstrate mastery. Rich tasks with multiple entry and exit points allow for natural differentiation of instruction and are accessible for all students.

The initial cluster at each grade includes a focus on **building mathematical community**. Learning mathematics involves productive struggle during problem-solving and meaningful discourse as students share strategies and explain their thinking. This requires individual students to have a mathematical mindset, a belief that they can learn and do mathematics, so they will take risks when solving non-routine tasks. Collectively, students must share ideas publicly as they critique mathematical ideas with peers and teacher. A safe community where mistakes and struggles are valued as learning opportunities is essential. Mathematical norms about how students do and talk about mathematics need to be explicitly established in the same way that other routines and expectations are introduced at the beginning of a school year.

NC.4.OA.3 is addressed on an ongoing basis throughout the year.

- Students should engage in **solving story problems daily**, not as a separate unit.
- Students continually reason and reflect on their work, which includes the use of estimation strategies.
- Representing problems using equations with a letter standing for the unknown quantity should also occur regularly, but may not be stressed during beginning exploration of new concepts.
- Interpretation of remainders in word problems should take place during work with division. Ensuring students continually monitor their ability to contextualize the numbers they compute will help make this a natural part mathematics.

Cluster 1: Building a Math Community through Real Data

Duration: 1-2 weeks

Content Standards:
This list includes standards that will be addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Please note strikethroughs and recommendations in the Important Considerations section for more information.

Represent and Interpret Data
NC.4.MD.4
Represent and interpret data using whole numbers.
- Collect data by asking a question that yields numerical data.
- Make a representation of data and interpret data in a frequency table, scaled bar graph, and/or line plot.
- Determine whether a survey question will yield categorical or numerical data.

Supporting Standards:
NC.4.NBT.4
Add and subtract multi-digit whole numbers ~~up to and including 100,000~~ using the standard algorithm with place value understanding.

Mathematical Practices:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?
The focus of this cluster is building an effective math environment. Representing and interpreting data will be the content used to begin the development of a classroom culture where students respect and value each individual's contribution to the classroom.

Consider the following elements when preparing for an effective math environment:
1.) Develop mathematicians with positive attitudes about **their** ability to **do** mathematics by:
- Creating opportunities to develop an appreciation for mistakes
- Seeing mistakes as opportunities to learn
- Teaching students to take responsibility for their learning

2.) Develop mathematicians who **respect others** by:
- Demonstrating acceptance, appreciation, and curiosity for different ideas and approaches
- Establishing procedures and norms for productive mathematical discourse
- Considering various solution paths

3.) Develop mathematicians with a **mindset for problem solving** by:
- Encouraging student authority and autonomy when problem solving
- Emphasizing questioning, understanding, and reasoning about math, **not** just doing math for the correct answer
- Asking follow-up questions when students are both right and wrong

- Allowing students to engage in **productive struggle**

During this cluster:

- Students will generate data by formulating a question(s) (Example: Favorite summer vacation, number of siblings, weight of backpack, how many minutes you are on a device in a week, favorite flavor of ice cream, etc.).
- Students will determine whether a survey question will yield **categorical data** (favorite summer vacation, favorite flavor of ice cream) or **numerical data** (number of siblings, weight of backpack, hours on device).
- Students will design a plan to collect and represent the data (ex: using frequency tables, scaled bar graphs, and/or line plots).
- Students will analyze and interpret their data.
- Students will apply computation skills when asking and answering questions about the data.

Important Considerations

- For success, significant time should be spent setting up the classroom. This includes:
  - Developing classroom norms for communication (ex: non-verbal signals, listening and speaking expectations, talk moves for math discussions).
  - Developing math routines (ex: number of the day, number talks, number strings, and other appropriate math routines).
  - Setting various expectations for the structure of the math block (ex: expectations for whole class instruction, cooperative learning, independent learning, etc.).
- Math discourse needs explicit modeling and practice. This includes students:
  - Sharing their thinking
  - Actively listening to the ideas of others
  - Connecting to others' ideas
  - Asking questions to clarify understanding
- In Grade 2, students solve simple put-together, take-apart, and compare problems in a data context using information presented in a picture and bar graph. In Grade 3, students solve one and two step 'how many more and how many less problems' using the information from graphs.
- In third grade, students add and subtract numbers up to and including 1,000. The numbers in this unit should be limited to numbers students have worked with as new content related to addition and subtraction will be the focus of Cluster 3.
- In NC.4.MD.4, the line plot is a new representation of data.
- Integrate the data standard throughout the year and across content areas when possible. Students should use relevant real-world data to make conjectures about activities related to the world around them. Take advantage of opportunities to incorporate data standards with Science and Social Studies.
- Both additive (ex. How many more fish does class A have than class B) and multiplicative (ex. Class A had three times as many fish as class B, vanilla ice cream had two times more votes than chocolate ice cream) comparisons can be made and asked based on the data. Students use the language of times as much or times as many to lay a foundation for the focus on multiplicative comparison in Cluster 2.

Cluster 2: Explore multiplicative comparison, area and perimeter, factors, and multiples

Duration: 3-4 weeks

Content Standards

This list includes standards that will be addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Please note strikethroughs and recommendations in the Important Considerations section for more information.

Represent and solve problems involving multiplication and division.
NC.4.OA.1
Interpret a multiplication equation as a comparison. Multiply or divide to solve word problems involving multiplicative comparisons using models and equations with a symbol for the unknown number. Distinguish multiplicative comparison from additive comparison.

Gain familiarity with factors and multiples.
NC.4.OA.4
Find all factor pairs for whole numbers up to and including 50 to:

- Recognize that a whole number is a multiple of each of its factors.
- Determine whether a given whole number is a multiple of a given one-digit number.
- Determine if the number is prime or composite.

NC.4.MD.3
Solve problems with area and perimeter.

- Find areas of rectilinear figures with known side lengths.
- Solve problems involving a fixed area and varying perimeters with a fixed perimeter and varying areas.
- ~~Apply the area and perimeter formulas for rectangles in real world and mathematical problems.~~

Use the four operations with whole numbers to solve problems.
NC.4.OA.3
Solve two-step word problems involving the four operations with whole numbers.

- Use estimation strategies to assess reasonableness of answers.
- Interpret remainders in word problems.
- Represent problems using equations with a letter standing for unknown quantity.

Mathematical Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

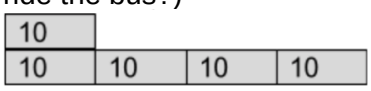
What is the mathematics?

In third grade, students focus on equal group and array multiplicative situations. In fourth grade, multiplicative thinking is expanded to include multiplicative comparison. Multiplicative comparison problems describe the relationship between two quantities, in which one is a multiple of the other (times as many, times more/less than). Data and scaled bar graphs from the previous cluster can be used as a context to represent and discuss both additive and multiplicative comparisons and to help students distinguish between the two. In fourth grade, students continue to develop the concept of area based on the array model and use that model to explore factors and multiples.

- Students will investigate and explain the difference between multiplicative comparisons and additive comparisons (ex: What is the difference between saying 3 times as many/3 times less than and 3 more/3 less than?).
- Students will explore multiplicative relationships through multiple contexts and models (ex: area model, tape diagram, scale bar graph, measurement situations, money situations).
- Students will solve both unknown product and unknown factor situations (ex: Keenan is 4 feet tall, the tree he planted is now 5 times taller than he is. How tall is the tree?; Keenan is 4 feet tall. The tree is 24 feet. How many times taller is the tree than Keenan? In both examples, students should draw a diagram to show the relationship between Keenan's height and the height of the tree.)
- Students will explore finding factors using the array/area model (ex: Find all the rectangles that can be made using exactly 24 tiles. For rectangles with an area of 24, the side lengths of the rectangles are factors of 24).
- Students will verify that some numbers can be made into more than one rectangular array (composite numbers) and some numbers can only be represented by rectangular arrays with 1 row (prime numbers); then use that understanding to define prime and composite numbers.
- Students will investigate and discover that a whole number is a multiple of each of its factors (ex: Make a rectangle to show that 4 is a factor of 24? How do you know?; How many hops of 6 on a number line does it take to get to 24? So, 24 is 4 times as many as six. What other equal hops could you make to reach 24?).
- Students will explore and compare fixed areas and fixed perimeters and formulate conclusions about the relationship between area and perimeter.

Important Considerations:

- The work in this cluster builds off the 3rd grade focus of multiplication as repeated addition and equal groups and includes a new interpretation in 4th grade of multiplication as comparison.
- Multiplicative comparison situations in this cluster should be limited to comparisons within 100 focusing first on building foundational language and conceptual understanding of this new interpretation.
- Placing this cluster early in the year will allow for time to review multiplication facts within 100 to further develop fluency.
- When exploring multiplicative comparison, the phrases *how many times more than/less than, how many times fewer than*, and *times as many* help connect the understanding that the comparison is based on one set being a multiplier of the other. Using a tape diagram (also known as bar model) can help develop this idea. (ex: If four times as many 3rd grade students ride the bus to school compared to the 10 that walk to school, how many students ride the bus?)



- Many rich contexts can be used to practice multiplicative comparison (ex. Building off data collection from cluster 1, if twice as many people ride in cars to school compared to walking; review customary measurement from grade 3 to discover that a quart is four times as much as a cup and twice as much as a pint; In pattern-finding, the number of blocks in a shape pattern can be 3 times the term number or the number of eyes in 6 dogs is twice as many as the number of eyes in 3 dogs).
- Multiplicative comparison can also be used in area and perimeter investigations (ex: A rectangle is 2 tiles wide. It is 5 times as long as it is wide. Create a model to find the rectangle's length; A dog kennel is 5 feet wide. It is 4 times as long as it is wide. What is the perimeter of the kennel?).
- There are many misconceptions related to the concepts of area and perimeter. It is important that formulas are not preceded by the understanding of each unique measure. Provide hands-on experiences, where students manipulate and measure shapes to discover students that it is possible to change the area of a figure without changing its perimeter.
- Students will investigate and explain that it is possible to have different rectangles with the same perimeter but different areas and vice versa (ex: Students investigate how many different rectangles can be made with 36 tiles. Find and record the perimeter of the rectangle. Also, using 24 cm of string, students investigate the different rectangles that can be made with the fixed perimeter. Find and record the area.).

Cluster 3: Use place value strategies to add and subtract whole numbers

Duration: 3-4 weeks

Content Standards

This list includes standards that will be addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Please note strikethroughs and recommendations in the Important Considerations section for more information.

Generalize place value understanding for multi-digit whole numbers.

NC.4.NBT.1
Explain that in a multi-digit whole number, a digit in one place represents 10 times as much as it represents in the place to its right, up to 100,000.

NC.4.NBT.2
Read and write multi-digit whole numbers up to and including 100,000 using numerals, number names, and expanded form.

NC.4.NBT.7
Compare two multi-digit numbers up to and including 100,000 based on the values of the digits in each place, using >, =, and < symbols to record the results of the comparisons.

Use place value understanding and properties of operations to perform multi-digit arithmetic.
NC4.NBT.4
Add and subtract multi-digit whole numbers up to and including 100,000 using the standard algorithm with place value understanding.

Use the four operations with whole numbers to solve problems.
NC.4.OA.3
Solve two-step word problems involving the four operations with whole numbers.

- Use estimation strategies to assess reasonableness of answers
- Interpret remainders in word problems
- Represent problems using equations with a letter standing for the unknown quantity.

<u>Supporting Standards:</u>
NC.4.OA.1
Interpret a multiplication equation as comparison. Multiply or divide to solve word problems involving multiplicative comparisons using models and equations with a symbol for the unknown number. Distinguish multiplicative comparison from additive comparison.

NC.4.MD.8
Solve word problems involving addition and subtraction of time intervals that cross the hour.

Mathematical Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?

In this cluster students extend their place value understanding from previous grades to include numbers up to the hundred thousands place by building on the work they have already done this year with multiplicative comparison. They then use this place value knowledge to add and subtract numbers within 100,000 using the standard algorithm in the context of one and two step word problems.

- Students increase their number sense about larger numbers with experiences estimating (ex. How many tennis balls fit in this shoe box? How many tennis balls would you estimate fit in our classroom? How much space would you need for 10,000 tennis balls? How many pages would it take to draw 100,000 stars?).
- Students build on their knowledge of multiplicative comparison from Cluster 2 to explore numbers that are 10 times greater or ten times less than other numbers. They use various tools (ex. calculator, zoomable number lines, place value blocks) to notice patterns when a number is continually multiplied or divided by ten to discover that a digit in one place is ten times as much as the digit to the right.
- Students name larger numbers within 100,000 with numerals, number names, and expanded form, connecting the expanded form to their explorations about each place being ten times more than the place to the right.
- Students use their knowledge of place value to compare two multi-digit numbers up to 100,000. They write those comparisons in number sentences with the greater than, less than, or equals sign.
- Students continue to solidify estimation, decomposition, and whole number strategies for addition and subtraction and use the relationship between addition and subtraction. As they add and subtract numbers within 100,000, they use their place value knowledge to understand the addition and subtraction algorithms as an efficient strategy to compute with larger numbers.
- Students work in multiple contexts with one and two-step word problems throughout the cluster. They use variables to represent unknown quantities in number sentences. They use estimation to anticipate and assess the reasonableness of their solutions.

Important Considerations:

- This cluster is the first point in Grade 4 where students work with larger numbers, since all Cluster 1 and 2 involve working with smaller numbers. Conceptual understanding of place value requires a foundation of multiplicative comparison and is the foundation of addition and subtraction with algorithms, so it should be thoroughly unpacked. Teaching place value at the beginning of the year before multiplicative comparison makes it difficult for students to gain more than a superficial understanding of just how to name large numbers.
- Place value and estimation strategies should be comprehensively incorporated into all work involving addition and subtraction.
- Students should understand that addition and subtraction operations still have the same meaning even with larger numbers. As they solve word problems in all the different problem types, note that the situations in which we add and subtract to solve have not changed.

- As students use variables and symbols to describe their work, be attentive to how the equal sign is being used. Avoid long strings of equations on the same line (ex. 2 + 2 = 4 + 5 = 9), which promote the idea that the equals sign means "the answer is coming." It is particularly important for students to see equals meaning "has the same value" to avoid misconceptions about the equal sign that often persist into formal high school algebra.
- Work with students to discuss when it is efficient to use different strategies to add and subtract. The addition and subtraction algorithms are efficient in some problems (particularly with large numbers), but in other problems, estimation, decomposition, compensation, or constant difference strategies might be more efficient (ex. 10,000 - 8,924 = 9,999 - 8,923). Fluency in mathematics is accuracy, efficiency, and flexibility.
- As students are making conjectures when looking for patterns as they multiply by ten, be sure to help them refine their mathematical language so that their statements are accurate. Students often notice that when they multiply a number by ten a zero is added to the end. This pattern is only true for whole numbers, however. This can create a misconception later as students begin working with decimals. Helping introduce counterexamples (ex. Does your "zero at the end" idea work with fractions?) can help students refine their statements. With experience they will begin to refine their own and each other's statements, gaining experience in Mathematical Practice 6, attending to precision.

Cluster 4: Develop multi-digit multiplication and division strategies through meaningful contexts and models

Duration: 5-6 weeks

Content Standards

This list includes standards that will be addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Please note strikethroughs and recommendations in the Important Considerations section for more information.

Use place value understanding and the properties of operations to perform multi-digit arithmetic.

NC.4.NBT.5

Multiply a whole number of up to three digits by a one-digit whole number, and multiply up to two two-digit numbers with place value understanding using area models, partial products, and the properties of operations. Use models to make connections and to develop the algorithm.

NC.4.NBT.6

Find whole-number quotients and remainders with up to three-digit dividends and one-digit divisors with place value understanding using rectangular arrays, area models, repeated subtraction, partial quotients, properties of operations, and/or the relationship between multiplication and division.

Solve problems involving area and perimeter.

NC.4.MD.3

Solve problems with area and perimeter.

- Find areas of rectilinear figures with known side lengths.
- Solve problems involving a fixed area and varying perimeters with a fixed perimeter and varying areas.
- Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Use the four operations with whole numbers to solve problems.

NC.4.OA.3

Solve two-step word problems involving the four operations with whole numbers.

- Use estimation strategies to assess reasonableness of answers.
- Interpret remainders in word problems.
- Represent problems using equations with a letter standing for unknown quantity.

Supporting Standards

Represent and solve problems involving multiplication and division.

NC.4.OA.1

Interpret a multiplication equation as a comparison. Multiply or divide to solve word problems involving multiplicative comparisons using models and equations with a symbol for the unknown number. ~~Distinguish multiplicative comparison from additive comparison.~~

NC.4.NBT.1

Explain that a multi-digit whole number, a digit in one place represents 10 times as much as it represents in the place to its right, up to 100,000.

Mathematical Practices:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?

During this cluster, students will apply their understanding of place value, the models for multiplication (array/area model, tape diagram, partial products, etc.), and the properties of operations (distributive, associative, commutative properties) to explore, develop, discuss and solve multiplication and division problems involving multi-digit numbers. A variety of contexts should be used to provide opportunities for students to select and make sense of different strategies to compute numbers. Investigation of area and perimeter should be incorporated throughout this cluster. Students will use their understanding of rectangles and reason about multiplication and division related to the dimensions of a rectangle. Through the investigations, the area and perimeter formulas will emerge in connection to the area model. Students will also explore division using the area model and the relationship between multiplication and division to solve problems with missing side measures. Students will expand on their work of representing and solving problems using the area model and partial products solving two-digit by two-digit multiplication problems. As the numbers get larger, students will see the need for more efficient strategies. Students will compare and connect the standard algorithm to the area model and partial products method.

- Students will explore multiplication and division patterns when multiplying/dividing by 10, 100, and 1,000 through various models (ex: place value blocks and disks, array model, tape diagram, numerically- 4 x 3, 4 tens x 3, 4 hundreds x 3).
- Students will use the properties of operations to decompose numbers (3 x 20 = 3 x 2 x 10).
- Students will use estimation and mental math strategies as an ongoing practice to calculate and reason about products and quotients.
- Students will use the area model, partial products, and other appropriate strategies to represent and solve multiplication problems.
- Students will use the area model and partial quotients to represent and solve division problems (both partitive and quotative division situations).
- Interpret division problems as either number of groups unknown or group size unknown.
- Students will understand the inverse relationship between multiplication and division.
- Students will explore multiple ways to divide multi-digit numbers and be able to explain chosen strategy.
- Students will describe the role of place value in multiplication and division strategies.
- Students will compare the area model, partial products, and the standard algorithm and understand the connection through use of the distributive property.
- Students will choose efficient and appropriate strategies to solve multiplication or division problems.

Important Considerations:

- Contexts that are familiar and relevant to students are a meaningful way to engage **all** students, in meaningful development of multiplication/division concepts, models, and relationships. Contexts can be developed using common classroom experiences, through read alouds, from a story and/or pictures, and written in a story problem format. Rich tasks

with multiple entry and exit points allow for natural differentiation of instruction and are accessible for all students.

- This cluster has many opportunities to make relevant mathematical connections between topics. (ex: Exploring multiplication and division patterns when multiplying/dividing by 10, 100, and 1,000 is an opportunity to further develop understanding of the base ten system; students make comparison, place value, and multiplication connections as the value of digits change when they are multiplied by 10, 100, and 1,000. (ex: 500 is 10 times greater than 50); Building on work with multiples, students break apart factor(s) into multiples of 10 and 100 before multiplying (3 x 20 = 3 x 2 x 10). Solve missing factor problems given a comparison situation and use multiplicative thinking or division to find the missing factor.)
- Allow students to model and solve problems in ways that make sense to them. Teachers may create a context to lead students toward a specific model, but honor divergent ideas and strategies as students engage in the math. Comparing different ideas and strategies to solving problems can lead students to deeper mathematical understanding.
- Students should solve division problems with/without remainders. Discussion about how to interpret remainders based on the context of the problem being solved is important as students decide whether they need to round up (ex. We have 20 students going on the field trip. Each car can carry 3 students. How many cars are needed?) or down (ex. I have $20 to buy packs of stickers. Each pack costs $3. How many packs can I buy?) or use the exact amount (ex. We have 20 brownies to share with 3 table groups. How many brownies will each table group get?)
- As the strategies for solving multiplication and division problems become more efficient, the math becomes more abstract (ex: going from the area model of 4 partial products → area model with 2 partial products → partial products without the area model → standard algorithm). Some students (specifically students with disabilities) may not be ready to use the standard algorithm *with understanding*. All students should continue to be exposed and work towards efficient, accurate strategies, but should not use procedural strategies without understanding.
- Students are not expected to be fluent with standard algorithm until 5th grade.
- Students work in multiple contexts with one and two-step word problems throughout the cluster. They use variables to represent unknown quantities in number sentences. They use estimation to anticipate and assess the reasonableness of their solutions.

Cluster 5: Extend Understanding of Fractions

Duration: 2-3 weeks

Content Standards

This list includes standards that will be addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Please note strikethroughs and recommendations in the Important Considerations section for more information.

Extend understanding of fractions.

NC.4.NF.1

Explain why a fraction is equivalent to another fraction by using area and length fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.

NC.4.NF.2

Compare two fractions with different numerators and different denominators, using the denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions by:

- Reasoning about their size and using area and length models.
- Using benchmark fractions 0, ½, and a whole.
- Comparing common numerator or common denominators.

Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?

This cluster extends fraction equivalence and ordering that was first introduced in Grade 3. In Grade 4, students will use visual models and explain why a fraction is equivalent with attention to how the number and size of parts differ even though the amounts themselves are the same. Students compare and order fractions through reasoning about the size of fractions using benchmark fractions, models, and/or the use of equivalent fractions. Students use greater than, less than, and equal to symbols to record comparisons and are able to justify their conclusions. Student continue the work of equivalent fractions and explore decimal fractions through the use of models. Students use their experience with fractions and place value to make connections between fractions and decimals.

- Students will expand their understanding of fractions (ex: numerator and denominator relationship to the whole) and verify their thinking with visual models (ex: area model, number line, tape diagram.
- Students will explain that fractions are relative to the size of the whole and apply that understanding when comparing fractions.

- Investigate equivalence through real world contexts.
- Represent equivalent fractions using models (area model, number line, tape diagram) and verify that while the number and the size of the parts differ, the fractional amount is the same.
- Use models/pictures/number lines to explain why different fractions are (or are not) equivalent.
- Recognize and generate equivalent fractions.
- Use benchmark fractions to reason about fractions including comparing and ordering fractions
- Develop multiple strategies (area models, number lines, benchmark fractions, and/or understanding of equivalence, using common numerators or common denominators) to compare and order fractions.
- Use understanding of equivalent fractions to explore decimal fractions (tenths and hundredths using area model, number line, decimal grids).

Important Considerations:

- Prior to beginning work with equivalent fractions, ensure that students have experience with multiple models to represent fractions and understand the connection between the symbolism of the fraction and the meaning of the numerator and denominator.
- As students develop an understanding of equivalent fractions through use of visual models, they begin to see the relationship between multiplication and fraction equivalence.
- Students need the chance to create their own drawings when working on tasks rather than always being presented with pre-made fraction materials that are already shared fairly for them.
- In fifth grade, students learn equivalence as “multiplying by 1.” Note that this is not the focus in fourth grade as the goal is for students to reason and justify their thinking using models.
- Using a context(s) when engaging in comparison can help students understand why it is only valid to compare two fractions when they refer to the same whole.

Cluster 6: Making Connections to Decimal Notation

Duration: 1-2 weeks

Content Standards

This list includes standards that will be addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Please note strikethroughs and recommendations in the Important Considerations section for more information.

Extend understanding of fractions.

Understand decimal notation for fractions and compare decimal fractions.

NC.4.NF.6

Use decimal notation to represent fractions.

- Express, model and explain the equivalence between fractions with denominators of 10 and 100.
- Use equivalent fractions to add two fractions with denominators of 10 or 100.
- Represent tenths and hundredths with models, making connections between fraction and decimals.

NC.4.NF.7

Compare two decimals to hundredths by reasoning about their size using area and length models, and recording the results of comparisons with the symbols >, =, or <. Recognize that comparisons are valid only when the two decimals refer to the same whole.

Mathematical Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?

In this cluster, students explore decimal numbers via their connection to decimal fractions. Students will express quantities using both fraction and decimal notation. Students apply the same reasoning to decimal numbers as developed through their fraction work.

- Students will represent tenths and hundredths with models and make connections between the meaning of fractions and decimals. (ex: Models may include a number line, decimal grids, tenth/hundredth circles; Make connections between fractions with denominators of 10 and 100 on place value chart).
- Students will use the correct language to describe decimals (ex: Say ‘three tenths’ instead of ‘point three’ when describing 0.3).
- Students will use models to verify equivalence (ex: use number line and show fraction on top and decimal notation on bottom) to express equivalence between fraction and decimal form (e.g. 3/10 = 0.3)
- Students will use decimal notation in reference to the number line and decimal grid models to solve problems and communicate their thinking.

- Students will compare decimal numbers using area models, number lines, benchmark numbers, and/or understanding of equivalence, to compare and order decimals.
- Students will understand that, like fraction notation, decimal comparisons are valid only when comparing to the same whole.
- Students will use greater than, less than, and equal symbols to record comparisons.

Important Considerations:

- Helping students make connections between mathematical ideas is important. Use concrete examples and models to connect students understanding of fractions to decimal fractions to decimal notation. (ex: In a context, discuss and show a visual model students are familiar with-- a chocolate bar, pizza, meter, etc. Discuss the meaning of the model related to numerator and denominator. Repeat modeling, but this time dividing the model into tenths. Students should identify one portion as 1/10. Explain that there is another notation for expressing tenths. Introduce notation and verbally express both representations as “one-tenth”)
- While the decimal point is a convention, it is based on understanding the structure of the place-value system. It is important for students to understand that the decimal point serves a very important purpose of separating whole numbers from parts (ex: dollars and cents, whole object from part of an object, scores/time, etc.).
- Measurement and money contexts provide students with concrete examples exploring tenths and hundredths that can support student understanding and reasoning about decimals.
- It is important to provide opportunities to explore various representations and forms of decimals (ex: 0.31 thirty-one hundredths is also 3 tenths and 1 hundredth; 0.30 can be described as 3 tenths or thirty-hundredths).

Cluster 7: Understanding operations of fractions and decimals

Duration: 4-5 weeks

Content Standards

This list includes standards that will be addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Please note strikethroughs and recommendations in the Important Considerations section for more information.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

NC.4.NF.3

Understand and justify decompositions of fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100.

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of unit fractions and a sum of fractions with the same denominator in more than one way using area models, length models, and equations.
- Add and subtract fractions, including mixed numbers with like denominators, by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- Solve word problems involving addition and subtraction of fractions, including mixed numbers by writing equations from a visual representation of the problem.

Use unit fractions to understand operations of fractions.

NC.4.NF.4

Apply and extend previous understandings of multiplication to:

- Model and explain how fractions can be represented by multiplying a whole number by a unit fraction, using this understanding to multiply a whole number by any fraction less than one.
- Solve word problems involving multiplication of a fraction by a whole number.

Understand decimal notation for fractions, and compare decimal fractions.

NC.4.NF.6

Use decimal notation to represent fractions.

- Express, model and explain the equivalence between fractions with denominators of 10 and 100.
- Use equivalent fractions to add two fractions with denominators of 10 or 100.
- Represent tenths and hundredths with models, making connections between fractions and decimals

Mathematical Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?

In this cluster, students build and on their understanding of unit fractions, relative size of fractions, and fraction equivalence to reason and compute with numbers. Students compose fractions from unit fractions and decompose fractions into unit fractions (including mixed numbers) representing their understanding with visual models and number sentences. Using fraction knowledge and prior understanding of whole-number operations, students explore and develop fraction computation strategies.

- Students will represent decompositions of fractions (including mixed numbers) using visual models (e.g. area model, number line, tape diagrams).
- Students will write a number sentence to match visual representations. (e.g. show ¾ using a model and write an appropriate number sentence matching the model, ¾ = ¼ + ¼ + ¼)
- Students will use repeated addition to describe decomposed fractions and connect that to their understanding of multiplication. (e.g. Show 3 skips of ¼ on a number line and connect that understanding to multiplication.)
- Students will describe decomposed fractions as a whole number times a unit fraction (¾ = ¼ + ¼ + ¼ = 3 x ¼).
- Students will build on meaning of whole number multiplication and decomposition of fractions to multiply a fraction less than one by a whole number (ex. ½ x 8 can be thought of as half a group of 8 or 8 groups of ½).
- Students will solve situations of fractions of a group by relating it to multiplication of a fraction by a whole number (e.g., 2/3 of the children are girls, 2/3 of 12 is the same as 2/3 * 12 = 8).
- Students will explore fraction operations (addition, subtraction, and multiplication) using a variety of models (ex: area model, number line, tape diagrams, set models)
- Students will use equivalent fractions and/or properties of operations to add and subtract fractions (including mixed numbers).
- Students will use visual models along with addition and multiplication to build fractions greater than 1.
- Students will understand that fractions greater than one can be described both as numerator greater than the denominator (improper fraction) and as a mixed number.
- Represent decompositions of a whole into tenths using visual models and both fraction and decimal notation (ex: area model/decimal square and number line).
- Use equivalent fractions to add two fractions with denominators 10 and 100.
- Solve addition of decimals problems using understanding of equivalence, concrete decimal models, and using decimal notation.

Important Considerations:

- In Grade 4, expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Use a variety of meaningful contexts to engage students in computational tasks without giving rules or procedures to solve them. Contexts do not need to be elaborate, but it is important to think about contexts that fit the operation and that is reasonable for the learning goal.
- Choose contexts that encourage different models (ex: a meter might encourage a linear model, a garden or quilt as an area model, etc.), use different types of numbers in the problems (whole, mixed, improper, fractions) and vary the operations so that students continue to make meaning first.
- Integrate previous topics such as multiplicative comparison word problems, data sets involving metric measurement, properties, time concepts, area and perimeter, etc.

- Estimation should be a part of the development of fraction computation to continue the focus of reasoning/reflecting on the expected results as well as focusing on the meaning of the operation. (ex: Over or under estimation routine: Ask students to estimate the sum or difference of two fractions. Thumbs up if over, thumbs down if under. Discuss thinking. Students should justify using a number line, area model, example, etc.)
- Students should be flexible in the ways they solve fraction computation problems. In order to be flexible, they need to understand what they are doing and why they are doing it.
- Algorithms for computing fractions do not help students think conceptually and should not be used as a way to construct understanding of the operations.
- As students are using visual models, assure they connect the models to the symbolic representation of the operations. For some students, this connection needs to be made explicit as they may see the representations as different math skills.

Cluster 8: Applying geometric concepts

Duration: 2-3 weeks

Content Standards

This list includes standards that will be addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Please note strikethroughs and recommendations in the Important Considerations section for more information.

Classify shapes based on lines and angles in two-dimensional figures.

NC.4.G.1

Draw and identify points, lines, line segments, rays, angles, and perpendicular and parallel lines.

NC.4.G.2

Classify quadrilaterals and triangles based on angle measure, side lengths, and the presence or absence of parallel or perpendicular lines.

NC.4.G.3

Recognize symmetry in a two-dimensional figure, and identify and draw lines of symmetry.

Understand concepts of angle and measure angles.

NC.4.MD.6

Develop an understanding of angles and angle measurement.

- Understand angles as geometric shapes that are formed wherever two rays share a common endpoint, and are measured in degrees.
- Measure and sketch angles in whole-number degrees using a protractor.
- Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.

Supporting Standards

Use the four operations with whole numbers to solve problems.

NC.4.OA.3

Solve two-step word problems involving the four operations with whole numbers.

- Use estimation strategies to assess reasonableness of answers.
- Interpret remainders in word problems.
- Represent problems using equations with a letter standing for unknown quantity.

Generate and analyze patterns.

NC4.OA.5

Generate and analyze a number or shape pattern that follows a given rule.

Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?

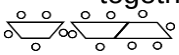
This cluster focuses on geometry and measurement. Students build spatial sense by noticing symmetry and attributes of two-dimensional shapes and using these attributes to identify them. They use tools to draw, sketch, and measure lines, angles, and shapes. Students build connections between geometry and number by generating and analyzing shape patterns to find a rule and by estimating and assessing the reasonableness of their measurements.

- Students use geometric vocabulary in explorations and tasks to identify points, lines, line segments, rays, angles, parallel and perpendicular lines (ex. Scavenger hunt in the school, on the playground; artwork; architecture; quilts; quick draw warm-ups, shape patterns, etc.)
- Students use rulers, protractors, and other tools (isometric drawing paper, patty paper, construction tools on apps, etc.) to draw lines, angles, and shapes.
- Students reason about the size of angles, using the right angle as a benchmark to determine if an angle is acute, obtuse, or right. They work on their number sense by reasoning about the size of an angle and making a reasonable estimate of its size (ex. A right angle is 90 degrees and this angle is about half way. It will be around 45 degrees; This angle is a little more than a right angle. My estimate is 100 degrees.)
- Students assess the reasonableness of their angle measurements using benchmark angles. (ex: A student uses a protractor to measure an angle to be 120 degrees. Student then uses benchmarks and notices that it is an acute angle and determines 120 degrees isn't reasonable.)
- Students solve for unknowns with angle measures. (A student noticed the clock hand rotated 45 degrees. If the next time they looked at the clock it formed a right angle, how far had it rotated between measures?)
- Students identify and draw right, obtuse, and acute angles.
- Students explore, identify, and draw lines of symmetry in 2-D figures.
- Students use attributes to identify quadrilaterals and triangles. Students create minimal defining lists (what the shape must have to be that shape) to distinguish between shapes that have much in common (ex. rhombuses, squares, and rectangles have two sets of parallel lines, but a square must have four congruent sides and four right angles).

Important Considerations:

- Memorizing names of shapes without understanding the attributes that define the shape can leave students with misconceptions. (ex: many students think that  is a triangle and that this  is not because it doesn't look like the triangle they had once identified.)
- In order for students to develop language with understanding, students need opportunities to explore (to see, to notice, and to discriminate between) shapes, to describe and categorize their results, to justify their thinking (ex: How do you know that is true? Is your thinking true for every triangle?), and make conjectures based on their thinking.
- Developing spatial sense and geometric reasoning can help prepare students for success in further mathematics. Spatial sense includes the ability to mentally visualize objects and special relationships. Specific spatial reasoning skills like mental rotation and visual spatial reasoning can be strengthened with meaningful experiences over the course of the year (ex: "quick image activity" the teacher flashes a design made with pattern blocks. Students see it for a couple seconds and then try to recreate the design with their own blocks. Create opportunities for students to build structures from Legos® or other blocks and challenge students with tangram puzzles and/or origami activities).
- Classroom instruction needs to include a variety of each type of shape (ex: triangles should include isosceles, scalene, and be shown in different orientations) to encourage students to focus on relevant properties rather than irrelevant properties.

- In fourth grade, the focus is on classifying and identifying quadrilaterals and triangles, but not the hierarchy of shapes. In the process of exploring attributes and sorting and identifying shapes, they may start to notice and discuss whether a shape can have more than one name (ex. A square is a rectangle). These conjectures can be discussed and explored, but students do not focus heavily on the hierarchy of triangles and quadrilaterals until fifth grade.
- Shape and number patterns provide rich contexts for students to use vocabulary, practice angle measures, develop spatial sense, and connect back to perimeter work from Cluster 2 (ex. As the number of sides increase, how does the sum of the angle measures increase? How many people can be seated at a single trapezoid table, two trapezoid tables pushed together, three trapezoid tables, ten trapezoid tables pushed together?)



- In addition to recognizing and identifying lines of symmetry, problem solving tasks involving symmetry provide a meaningful opportunity to engage in reasoning and communication of thinking. (ex: Create a design with pattern blocks that has exactly two lines of symmetry. Three lines of symmetry.; What design can you make that has more than 4 lines of symmetry? Explain how you determined your design has more than 4 lines of symmetry.)

Cluster 9: Using place value to understand metric measurement

Duration: 2-3 weeks

Content Standards:

This list includes standards that will be addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Please note strikethroughs and recommendations in the Important Considerations section for more information.

Solve problems involving measurement.

NC.4.MD.1

Know the relative sizes of measurement units. Solve problems involving metric measurement.

- Measure to solve problems involving metric units: centimeter, meter, gram, kilogram, liter, milliliter.
- Add, subtract, multiply, and divide to solve one-step word problems involving whole-number measurements of length, mass, and capacity that are given in metric units.

NC.4.MD. 2

Use multiplicative reasoning to convert metric meters from a larger unit to a smaller unit using place value understanding, two-column tables, and length models.

NC.4.MD.8

Solve word problems involving addition and subtraction of time intervals that cross the hour.

NC.4.NF.6

Use decimal notation to represent fractions (second two bullets only)

NC.4.NF.7

Compare two decimals to hundredths by reasoning about their size using area and length models, and recording the results of comparisons with the symbols >, =, or <. Recognize that comparisons are valid only when the two decimals refer to the same whole.

NC.4.OA.5

Generate and analyze a number ~~or shape pattern~~ that follows a given rule.

<u>Supporting standards:</u>

NC.4.MD.3

Solve problems with area and perimeter.

- Find areas of rectilinear figures with known side lengths.
- Solve problems involving a fixed area and varying perimeters with a fixed perimeter and varying areas.
- Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

NC.4.MD.4

Represent and interpret data using whole numbers.

- Collect data by asking a question that yields numerical data.
- Make a representation of data and interpret data in a frequency table, scaled bar graph, and/or line plot.
- ~~Determine whether a survey question will yield categorical or numerical data.~~

Mathematical Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?

In this cluster, students explore metric measurement for length, volume, and mass to develop a sense of the value of the different metric units. Students continue to build on the patterns and relationships when you multiply by 10 and 100 that were noticed when unpacking the place value system in Cluster 3. They apply their knowledge of multiplicative comparison to make sense of tenths and hundredths in the place value system and to convert from one metric unit to another using a conversion table. They work within the contexts of real-world data collection and measurement to represent and compare decimals.

- Students engage in multiple experiences measuring length, mass, and volume to gain a sense of the value of metric units (ex. About how much is a decimeter, a centimeter, a millimeter, a gram, a kilogram, a liter, a milliliter? Find ten objects in the room less than a decimeter strip and ten things more than a decimeter; How much space does a kilogram of cotton balls take up? A kilogram of dry beans? How many meters long is the hallway, school walking path?).
- As students measure and explore, they make estimates, assess the reasonableness of those estimates during the process of measuring and make adjustments.
- Students continue to build their understanding of multiplicative comparison and place value by measuring the same objects with different units and noticing the patterns in their measurements as ten times more or ten times less than the next unit (ex. The length of our table is 2 meters, 20 decimeters, 200 centimeters, 2000 millimeters.).
- Students generate and analyze number patterns in tables to support conversions among units (ex. One liter is the same value as 1,000 milliliters; two liters is the same value as 2,000 milliliters; three liters is the same value as 3,000 milliliters. How many milliliters in 8 liters?).
- Students use the meter stick as a length model to further explore decimal place values (ex. Roll dice and line up base ten blocks against a meter stick to “Race to a Meter” recording centimeter cubes as hundredths and decimeter cubes as tenths.).
- Students compare decimals to hundredths using length and area models (ex. Who jumped the farthest? Whose race time was the fastest?).
- Students solve word problems within the context of measurement explorations, science experiments, real life situations using metric units (ex. What is the area of our garden bed? What is the perimeter of the carpet? How many liters of water do we need for the field trip? If we want to double the recipe, how many grams of sugar do we need?).

Important Considerations:

- This cluster culminates the year by extending the ideas of multiplicative comparison (Cluster 2) and place value (Cluster 3) to make sense of tenths and hundredths.
- Data collection and analysis standards from the beginning of the year (Cluster 1) can be reviewed as students collect and display their measurements (numerical data) using graphs.
- One- and two-step word problems using all four operations can be presented in the context of measurement while working within the same unit. This does not include operations with decimals which are introduced in Grade 5. This cluster wraps up Grade 4, incorporates prior

work with all four operations, and provides opportunities for two-step problem solving and place value conversations.
- When engaging in tasks and word problems, students may review the interpretation of remainders from Cluster 5 (ex. It takes 2 meters of ribbon to do the bow for each present. I have 9 meters of ribbon left. How many presents can I wrap?).
- Students use a variety of tools to explore measurements including meter sticks, meter wheels, tape measures, rulers, balance scales, spring scales, graduated cylinders, and beakers. They may also other models (ex. a ten-stick from the base-ten blocks as a decimeter).
- Note that the decimeter is not listed in the standard as one of the units students need to know. However, it is helpful to include the decimeter as a tenth of a meter and ten times a centimeter to as a visual model of decimal place values.
- The only work that includes conversions is from larger units to smaller units within the same system (e.g., meters to centimeters, not centimeters to meters.).
- As students are first noticing number patterns and working with conversion tables they may add multiple times recursively (down the table) to find the conversion. Encourage students to reason explicitly (across the table) to find a conversion rule.