Kindergarten Instructional Framework

Cluster 1	**Building A Mathematical Community through Exploring Attributes**
Cluster 2	**Understanding the Relationship between Numbers and Quantities**
Cluster 3	**Comparing quantities with counting and spatial relationships**
Cluster 4	**Identifying, Describing, Classifying, and Composing Shapes**
Cluster 5	**Number Relationships between and among 1-10**
Cluster 6	**Exploring parts and wholes with joining and separating**
Cluster 7	**Foundations of Place Value - Exploring numbers 11-20**

Introduction

The purpose of this document is to connect and sequence mathematical ideas to enable teachers to plan learning opportunities for students to develop a coherent understanding of mathematics. **Clusters** and sequencing are designed to foster students' meaning making of the connections among mathematical ideas and procedures. This meaning making occurs over time. Therefore, the concepts are included in multiple clusters with increasing depth. They build across the year beginning with conceptual understanding and moving toward procedural fluency.

Each cluster includes a list of related **content standards** and a range of **suggested duration**. Standards indicate the mathematics expectations of students by the end of the school year. Standards are introduced and developed throughout the year, so the fact that a content standard is listed in a particular cluster does not indicate that it is to be mastered in the cluster. In some clusters, strikethroughs in the content standards denote the portion of the standard that will be taught later. In other clusters, the full standard appears, but suggestions about the intended focus are noted in the cluster descriptions. Because standards may be included in clusters long before mastery is expected, formative assessment is an essential tool for instructional planning and reporting student progress. This assessment naturally occurs as teachers elicit students' mathematical thinking and reasoning while doing mathematics.

Particular **Standards for Mathematical Practice** are indicated for each cluster. The listed suggestions are a guide for teachers. While the practices listed may lend themselves particularly well to the cluster's content, this does not imply that they are the only practices students will use. Students doing rich mathematical tasks will naturally engage in many mathematical practices as they do mathematics. During instruction teachers may observe and decide to highlight the other practices students are using beyond those listed in the cluster.

Each cluster includes a section called **"What is the mathematics?"** that describes the significant concepts and connections within the standards necessary for students to make sense of and use the mathematics. A second section called **"Important Considerations"** provides guidance based on student learning progressions as well as ideas and models for teaching within problem-solving situations. Problem-solving and mathematical reasoning define

what it means to do mathematics. Rich tasks (including word problems) provide students with concrete contexts to use as they are introduced to new mathematics. Later, work within such tasks allows students to develop understanding and eventually to demonstrate mastery. Rich tasks with multiple entry and exit points allow for natural differentiation of instruction and are accessible for all students.

The initial cluster at each grade includes a focus on **building mathematical community**. Learning mathematics involves productive struggle during problem-solving and meaningful discourse as students share strategies and explain their thinking. This requires individual students to have a mathematical mindset, a belief that they can learn and do mathematics, so they will take risks when solving non-routine tasks. Collectively, students must share ideas publicly as they critique mathematical ideas with peers and teacher. A safe community where mistakes and struggles are valued as learning opportunities is essential. Mathematical norms about how students do and talk about mathematics need to be explicitly established in the same way that other routines and expectations are introduced at the beginning of a school year.

Cluster 1: Building A Mathematical Community through Exploring Attributes

***Duration**:* 2-3 weeks

Content Standards:

This list includes standards addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Note strikethroughs and recommendations in the Important Considerations section for more information.

NC.K.MD.1
Describe measurable attributes of objects; and describe several different measurable attributes of a single object

NC.K.MD.2
Directly compare two objects with a measurable attribute in common, to see which object has "more of/less of" the attribute ***[without counting]*,** and describe the difference.

NC.K.MD.3
Classify objects into given categories; ~~count the numbers of objects in each category and sort the categories by count.~~

NC.K.G.1
Describe objects in the environment using names of shapes, and describe the relative positions of objects using positional terms.

Mathematical Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the Mathematics?

In this cluster we are establishing a foundation for all mathematical work by creating a positive and respectful climate for learning. The goal is to set up a classroom in which students will feel safe to engage in discourse around mathematical topics. Mathematical discourse can reveal understandings and misunderstandings, support robust learning by boosting memory, support deeper reasoning, support language development, and support development of social skills. In addition, building a community of learners with a mathematical mindset in which students persevere and learn from mistakes is essential.

- Students participate in discussions focused on exploring attributes. Students learn to share their thinking, listen to the ideas of others, and ask questions to clarify their own understanding.
- Students describe attributes and compare objects to see how they are the same and different (ex. quantity, size, shape, color, texture) with words like longer/shorter, more/less, bigger/smaller, heavier/lighter, softer/harder, etc.
- Students recognize that things can be alike and different in many ways. They sort objects and explain the attribute(s) used to sort.
- Students use shape vocabulary informally in block centers and math centers to talk about attributes of different shapes and how they are alike and different. Positional vocabulary is modeled in natural settings (ex. Bill is standing behind Susan. Amy is first in line).

• Students get to know each other by posing questions and collecting data about themselves and their surroundings (ex. how we get to school, lunch choice, types of shoes we are wearing)
Important Considerations: • Although neither counting nor geometry is the focus of this cluster, some students may naturally count in situations of sorting, describing, and comparing attributes. This is a good time to formatively assess both students' counting and their understandings of geometry as they are working (ex. In the block center, "How many cubes did you use in your tower?" or in a math center, "How many green triangles did you use to fill up that shape?" or "How many buttons in your pile were red?") • Formal development of geometry vocabulary is not a focus in this cluster. Rather students use terms informally through math centers and activities (blocks, sorting pattern blocks, etc.) NC.K.G.1 can be considered a supporting standard in this cluster. This standard will be address formally in Cluster 4. • Much of the time spent in this cluster will be setting up math routines and expectations about what it means to think and talk mathematically in your classroom.

Cluster 2: Understanding the Relationship between Numbers and Quantities

***Duration**:* 5-6 weeks

Content Standards:

This list includes standards addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Note strikethroughs and recommendations in the Important Considerations section for more information.

NC.K.CC.1

Know number names and recognize patterns in the counting sequence by:

- Counting ~~100 by ones.~~ Rote sequence to 20.
- ~~Counting to 100 by tens.~~

NC.K.CC.2

Count forward beginning from a given number within the known sequence, instead of having to begin at 1.

NC.K.CC.3

Write numbers from ~~0 to 20~~ (0-5 and then 6-10). Represent a number of objects with a written numeral ~~0-20~~ (0-5 and then 6-10), with 0 representing a count of no objects.

NC.K.CC.4

Understand the relationship between numbers and quantities.

- When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object (one-to-one correspondence).
- Recognize that the last number named tells the number of objects counted regardless of their arrangement (cardinality).
- State the number of objects in a group, of up to 5 objects, without counting the objects (perceptual subitizing).

NC.K.CC.5

Count to answer "~~H~~How many?" in the following situations:

- Given a number from ~~1-20~~ (1-10), count out that many objects.
- Given up to ~~20~~ 10 objects, name the next successive number when an object is added, recognizing the quantity is one more/greater.
- Given ~~20~~ 10 objects arranged in a line, a rectangular array, and a circle, identify how many.
- Given ~~10~~ 5 objects in a scattered arrangement, identify how many.

NC.K.MD.1

Describe measurable attributes of objects; and describe several different measurable attributes of a single object

NC.K.MD.3

Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

NC.K.G.3

Identify squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres as two-dimensional or three-dimensional.

Mathematical Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. **Model with mathematics**
5. **Use appropriate tools strategically**
6. **Attend to precision**

7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the Mathematics?

The mathematical discourse established in Cluster 1 should continue to be embedded and utilized throughout each successive cluster.

Students learn key concepts related to counting including:

- Forward rote number sequence
- One-to-one Correspondence
- Cardinality
- Keeping Track
- Concept of Zero

Students perceptually subitize the number of objects in a group of up to five objects.

- Perceptual Subitizing is the ability to "recognize" a small number of objects and know how many there are without counting (for example, recognizing the pattern arrangement on dice instantly without having to count the dots).
- Subitizing is a tool for building a sense of quantities and thinking in units other than one which facilitates computation and an understanding of additive and multiplicative reasoning.
- Subitizing is a fundamental skill in the development of students' understanding of number and determines a student's success in mathematics. Subitizing is initially introduced in at the beginning of kindergarten and will continue to be integrated all year.

Important Considerations:

- Cluster 2 continues the work of Cluster 1 by having students continue to describe and sort objects by attributes, but now students count the objects sorted (ex. How many are red? How many are blue? How many cylinders in your tower? How many cones?) Include two-and three-dimensional shapes in this sorting as students continue to use geometric language informally during centers and tasks (NC.K.G.1 can be considered a supporting standard in this cluster. This standard will be address formally in Cluster 4).
- In Cluster 3 students will compare the objects in their sorting (ex. Did you use more hexagons or squares on your pattern block puzzle?), but as students count, sort, play games, they may start to use terms like *more, less*, and *the same* in their conversations.
- Number ranges listed in the standards (rote counting to 100 and understanding relationships between numbers and quantities to 20) are goals to be mastered by the end of the year. At this time of year, students are focused first on the numbers 0-5 and then 6-10.
- Note that depending on experience, some students will come in already knowing these number ranges while others may need substantial time on 0-5. Many activities can be differentiated simply by changing the number ranges so that all students can move forward from the point at which you assess them during your observations in Cluster 1.
- Counting and perceptual subitizing should be taught concurrently, not linearly as listed above. Instead, as students have ongoing experiences with dot cards, pips on dice, pips on dominoes, rekenreks, holding up fingers, five frames, etc., they gain a sense of "three"-ness or "four"-ness, and no longer need to count by ones within 5.
- When working on counting objects to 10, work should first focus on counting a collection of given objects to determine how many. It is often useful to use geometric shapes as collections as a way to continue exposing students to geometric vocabulary and attributes. Next students should work to produce a collection of a given quantity. Students will work to relate the oral word, set of objects/image, and symbol, connecting one representation to another in any direction.
- The target for this cluster is saying the forward rote sequence to 20 by ones, but this number should not be a limit. Meaningful practice of the counting sequence should be part of daily

activities in the classroom. This skill should be introduced at the beginning of the year and continued throughout the year. Listed below are general benchmarks for this standard.

- Beginning of the Year: Count to 20
- By the Middle of Year: Count to 50
- By the End of Year: Count to 100

Cluster 3: Comparing Quantities with Counting and Spatial Relationships

Duration: 3-4 weeks

Content Standards:

This list includes standards addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Note strikethroughs and recommendations in the Important Considerations section for more information.

NC.K.CC.1
Count to ~~100~~ 50 by ones and by tens.

NC.K.CC.3
Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20, with 0 representing a count of no objects.

NC.K.CC.4
Understand the relationship between numbers and quantities.

- When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object (one-to-one correspondence).
- Recognize that the last number named tells the number of objects counted regardless of their arrangement (cardinality).
- State the number of objects in a group, of up to 5 objects, without counting the objects (perceptual subitizing).

NC.K.CC.5
Count to answer "How many?" in the following situations:

- Given a number from 1-20, count out that many objects.
- Given up to 20 objects, name the next successive number when an object is added, recognizing the quantity is one more/greater.
- Given 20 objects arranged in a line, a rectangular array, and a circle, identify how many.
- Given 10 objects in a scattered arrangement, identify how many.

NC.K.CC.6
Identify whether the number of objects, within 10, in one group is greater than, less than, or equal to the number of objects in another group, by using matching and counting strategies.

NC.K.MD.2
Directly compare two objects with a measurable attribute in common, to see which object has "more of/less of" the attribute and describe the difference.

NC.K.G.1
Describe objects in the environment using names of shapes, and describe the relative positions of objects using positional terms.

Mathematical Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?

Students continue to develop fluency in subitizing and counting through meaningful counting experiences that include spatial relationships. They begin to use numerals to identify and compare quantities up to 10.

The mathematical discourse established in Cluster 1 should continue to be embedded and utilized throughout each successive cluster.

Students compare quantities by:

- Counting a quantity in a set and comparing it to a quantity in another set (more, less, the same)
- Comparing a given set to 5 or 10 (Is it more than 5, less than 5, or the same as 5?)
- Comparing by matching objects (My tower of 8 cubes is taller than your tower of 3 cubes so there are more cubes.)

Students continue to develop the forward rote sequence through daily experiences. The target for this cluster is saying the forward rote sequence to 50 by ones and tens, but this number should not be a limit. Meaningful practice of the counting sequence should be part of daily activities in the classroom. This skill should be introduced at the beginning of the year and continued throughout the year. Listed below are general benchmarks for this standard.

- Beginning of the Year: Count to 20
- By the Middle of Year: Count to 50
- By the End of Year: Count to 100

Students continue to develop informal geometric language about shapes (both 2-D and 3-D) as part of their sorting and comparing. Shapes will be formalized in Cluster 4.

Important Considerations:

- In Cluster 1 students describe objects based on attributes. In Cluster 2, they counted objects. In this cluster students begin comparing quantities. Some students may still be solidifying counting concepts while others may have started comparing with counting at the end of Cluster 2.
- Providing experiences within rich measurement, data, and geometry tasks and centers allows for differentiation and deepening of understanding as students count and compare with many different representations and within many different contexts (Ex. Whose block tower is the tallest? Did we use more cylinders or cubes in our tower? Did more people get lunch in the cafeteria or bring lunch from home?). Through these contexts students continue to build on NC.K.MD.1 and NC.K.MD.3 from Clusters 1 and 2 and to informally build a foundation for NC.K.G.1 in Cluster 4.
- Continuing to work with spatial representations of numbers are essential to support students' development of subitizing and thinking in collections rather than counting by ones.
- Symbols for addition, subtraction, equals, greater than, and less than should not be introduced until the end of the year or even first grade. Instead use the words *more*, *less*, and *the same*. The purpose of symbols is to communicate relationships among numbers without context (ex. being able to say 3 + 4 = 7 or 4 > 3 does not rely on knowing what the things are that are being put together or compared). In Kindergarten, students work first to internalize these relationships in context (ex. three birds joining two birds on a branch; three hexagon pattern blocks and two triangles on my picture; a tower of four cubes is taller than a tower of three cubes). They describe the relationships with language (ex. 3 and 2 is 5; 4 is more than 3) and by labeling pictures (ex. In shake and spill with 5 two-color counters, color 3 red and label with three; color two yellow and label with 2).
- Students need time to explore the meaning of equals as "has the same value" rather than "the answer is coming" to avoid misconceptions about the equals sign that often persist into formal

high school algebra. When writing, students use the word “is” to denote equal (ex. 4 is 3 and 1).
- Students begin counting by ones up to 50 and begin work on counting by tens using ten frames.

Cluster 4: Identifying, Describing, Classifying and Composing Shapes

Duration: 3-4 weeks

Content Standards:

This list includes standards addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Note strikethroughs and recommendations in the Important Considerations section for more information.

NC.K.CC.3
Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20, with 0 representing a count of no objects.

NC.K.CC.5
Count to answer "How many?" in the following situations:

- Given a number from 1-20, count out that many objects.
- Given up to 20 objects, name the next successive number when an object is added, recognizing the quantity is one more/greater.
- Given 20 objects arranged in a line, a rectangular array, and a circle, identify how many.
- Given 10 objects in a scattered arrangement, identify how many.

NC.K.CC.6
Identify whether the number of objects, within 10, in one group is greater than, less than, or equal to the number of objects in another group, by using matching and counting strategies.

NC.K.G.1
Describe objects in the environment using names of shapes, and describe the relative positions of objects using positional terms.

NC.K.G.2
Correctly names squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres regardless of their orientations or overall size.

NC.K.G.3
Identify squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres as two dimensional or three-dimensional.

NC.K.G.4
Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, attributes and other properties.

NC.K.G.5
Model shapes in the world by building shapes from components and drawing shapes.

NC.K.G.6
Compose larger shapes from simple shapes.

NC.K.MD.1
Describe measurable attributes of objects; and describe several different measurable attributes of a single object

NC.K.MD.3
Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

Mathematical Practices:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision

7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the Mathematics?

The mathematical discourse established in Cluster 1 should continue to be embedded and utilized throughout each successive cluster.

- Shapes have attributes that are the same and different. We can talk about these similarities and differences with words and numbers. Some of these differences help us know what to call each shape (ex. number of sides). Other differences do not change the name of the shape (ex. color, size, orientation).
- Students recognize and correctly name various representations of the same shape (tan and blue pattern blocks are both rhombus; triangles that are non-regular in addition to the green pattern block triangle).
- Students find shapes in their environment by noticing attributes of real-world objects as curvy or straight or like a circle (ex. The column on the building is like a cylinder).
- As students work on writing numerals, they can notice that some have curvy lines and others only straight lines. Some face one way and some another way (directionality), but other attributes don't change the numeral (ex. blue marker, teeny-tiny, etc.). The numerals still represent the same quantity.
- Students distinguish between two-dimensional (flat) and three- dimensional (solid) shapes.
- Students use informal language to discuss the attributes and properties (ex. curvy, straight, long, short, number of sides, number of angles).
- Students compose and decompose shapes by working with blocks, cubes, pattern blocks, and through drawings. Students model 2-D and 3-D shapes (ex. using playdough to build a cube, cone, cylinder) including shape they see in the world (ex. using blocks to build a replica of a famous building).

Important Considerations:

- Students have already completed significant work on K.G.1 coming into this cluster through their experiences in Clusters 1-3 including sorting activities and using positional words in natural settings.
- Students should continue to work on counting and comparing by using numbers to describe mathematical situations (ex. the number of triangles used to cover pattern block puzzle, the number of circles found on a shape hunt). These experiences will allow students opportunities to solidify concepts of counting and comparing before moving to number relationships in Cluster 5.
- Students communicate precisely by engaging in discussion about their reasoning using informal geometric language. Student are not expected to memorize definitions of shapes; rather they become increasingly more precise through use of the terms as they describe, sort, and compare and contrast various 2-D and 3-D shapes.
- Students need many experiences to recognize shapes that are not in the typical orientation (ex. an upside-down triangle or a triangle that is not regular). Looking at examples and non-examples and sorting can help students begin to recognize shapes and their attributes.
- Concrete experiences are essential in developing spatial reasoning. Students need to be able to manipulate objects to explore orientation, positionality, and composition.
- Experiences should go beyond the standard set of pattern and attribute blocks found in math kits. Special attention should be paid to instructional materials such as posters and literature used in the classroom as some may display inaccurate or imprecise examples of shapes. In addition, manipulatives like patterns blocks are 3-D shapes that are often used and named as models of 2-D figures. Discussions as students are sorting, modeling, and drawing provide an opportunity for students to make sense of the difference between 2-D and 3-D figures.

Cluster 5: Number Relationship between and among 1-10

Duration: 4- 6 weeks

Content Standards:
This list includes standards addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Note strikethroughs and recommendations in the Important Considerations section for more information.

NC.K.CC.1
Know number names and recognize patterns in the counting sequence by:
- Counting 100 by ones.
- Counting to 100 by tens.

NC.K.CC.2
Count forward beginning from a given number within the known sequence, instead of having to begin at 1.

NC.K.CC.5
Count to answer "How many?" in the following situations:
- Given a number from 1-20, count out that many objects.
- Given up to 20 objects, name the next successive number when an object is added, recognizing the quantity is one more/greater.
- Given 20 objects arranged in a line, a rectangular array, and a circle, identify how many.
- Given 10 objects in a scattered arrangement, identify how many.

NC.K.CC.6
Identify whether the number of objects, within 10, in one group is greater than, less than, or equal to the number of objects in another group, by using matching and counting strategies.

NC.K.CC.7
Compare two numbers, within 10, presented as written numerals.

NC.K.OA.1 Represent addition and ~~subtraction~~, within 10:
- Use a variety of representations such as objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, or expressions.
- Demonstrate understanding of addition and ~~subtraction~~ by making connections among representations.

NC.K.OA.3
Decompose numbers less than or equal to 10 into pairs in more than one way using objects or drawings, and record each decomposition by a drawing or expression.

NC.K.OA.4
For any number from 0 to 10, find the number that makes 10 when added to the given number using objects or drawings, and record the answer with a drawing or expression.

NC.K.OA.6
Recognize and combine groups with totals up to 5 (conceptual subitizing).

Mathematical Practices:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

8. Look for and express regularity in repeated reasoning.

What is the Mathematics?

The mathematical discourse established in Cluster 1 should continue to be embedded and utilized throughout each successive cluster.

Students work with key number relationships including:

- Number ranges listed in the standards (rote counting to 100 and understanding relationships between numbers and quantities to 20) are goals to be mastered by the end of the year. In Cluster 2, the focus was first on the numbers 0-5 and then 6-10. In this cluster, students will build on this understanding to rote count to 100, count with objects up to 20, and understand number relationships to 10. Note that depending on experience, some students may still need time on counting and comparing objects 0-10. Many activities can be differentiated simply by changing the number ranges so that all students can move forward from the point at which you assess them during your observations in previous clusters.
- *Conceptual subitizing* provides a context for exploring number relationships, the basis for addition and subtraction. This standard leads directly to the first grade standard of composing and decomposing numbers 0 - 10 fluently.
    - Students visually see subgroups of quantities within a larger quantity and learn that the subgroups can be combined to compose a whole (ex. I saw a group of 4 dots and 2 dots on the card. There are six dots!)
- *Conceptual* subitizing involves not only recognizing the subgroups, but also combining them to compose a whole.
    - Students understand pattern arrangements of numbers as in subitizing activities to discover properties and concepts such as conservation, compensation, unitizing, (such as counting using numbers other than one, e.g., 2s, 5s, 10s) counting on, composing and decomposing numbers, as well as an understanding of arithmetic and place value.
- Students develop an understanding that each successive number name refers to a quantity that is one greater in value than the previous number (inclusion). They begin to count on from numbers other than one (ex. What comes before 8? 7 and 6 and 5 and 4 and 3 and 2 and 1; What comes after 8? 9 and 10.)
- By giving students varied experiences with counting/creating sets and counting forward from a given number, students are able to tell the total if one more were added or if one were taken away (one more/one less) (ex. Showing dot card with six dots. “I wish I had 7, how many more dots do I need?”; “I have 6 chips and I want to give one away, how many would I still have?” When shown a ten frame with 7 dots, students are able to explain how many when adding one more dot or taking 1 dot away.).
- Students will build upon their subitizing skills to compose and decompose benchmark numbers 5 and 10 (benchmark numbers: 5 & 10). This is an important step that will lead to a deeper understanding of joining and separating in Cluster 6. Students may learn to visualize 5 and 10 as anchor numbers through the use of the five frames and 10 frames. (ex. Using 5 and 10 frames or a rekenrek-- 6 is 5 and 1 more, 7 is 5 and 2 more, 7 is also three less than 10). Students build on conceptual subitizing and other number relationship experiences to explore breaking a quantity into parts and examining whether the total amount is still the same(part-part-whole) (ex.“My toothpick picture has three at the top and 2 at the bottom. There are 5 toothpicks so 5 can be three and two!”; One student shows 8 on their ten frame with 5 dots on the top row and 3 dots on the bottom row. Another student shows 8 with 4 dots on the top row and 4 dots on the bottom row. “There are 8 dots on both of our ten frames!”).
- Students develop an understanding that numbers can be decomposed in different ways and still have the same amount (sense of equality) (ex. Continuing from the part-part-whole

example above —My toothpick picture has 4 and 1 and yours has 3 and 2 and his has 1 and 1 and 1 and 1. They are all the same as five. Let's try them on the number balance to see if they are really the same.).

Students compare quantities by:

- Students will additionally compare quantities within 10 presented as written numerals. (This is a continuation of the comparisons of quantities completed in cluster 3.)

Important Considerations:

- At this point in the learning sequence, students should be able to quickly recognize small groups of numbers (0-5) in a variety of arrangements (perceptual subitizing). We now need to move forward from perceptual subitizing to conceptual subitizing to help students understand that smaller groups of numbers can be joined to make a larger number.
- The emphasis of **K.OA.1** in this cluster is for students to *represent* the addition situations of putting together and adding to. Students may record addition situations as concrete (cubes, counters, etc.) or often pictorial (drawings, pictures, etc.) representations. Subtraction will be addressed later. During this exploration of addition, it is important to remember to solve addition situations within the range 0-5.
- Students should continue to use geometry and measurement as contexts for number relationships (ex. Find a collection of shapes that would have 10 sides. [1 square and 2 triangles or 1 hexagon and 1 square]; Find two objects that would be as long as 8 cubes.). This will build on the foundation established in Clusters 1-4 (NC.K.MD.1, NC.K.MD.2.NC.K.MD.3, NC.K.G.1, NC.K.G.2, NC.K.G.6).
- Students should have varied representations of composing and decomposing numbers 5 and 10 (snap cubes, shape puzzles, toothpick pictures, five and ten frames, dot cards, rekenreks, pattern block puzzles). In addition, they can make observations in the classroom or world (ex. At my table, 3 people have velcro shoes and 2 people have tie shoes).
- When discussing the subject of equality teachers should take care to use the concept of equals to mean "has the same value as" instead of a cue to do an operation.
- No symbols, other than numerals, are expected to be used at this point by students. It is important to remember that very young children do not need to understand the symbols +, -, or = to begin to learn how to compose and decompose numbers. Symbols for addition, subtraction, and equals should be introduced only after students have internalized the number relationships and when the vocabulary of those relationships (more, less, and, is, etc.) is freely used. This typically occurs at the end of the year or even in first grade.

Cluster 6: Exploring parts and wholes with joining and separating

***Duration**:* 4 - 6 weeks

Content Standards:
This list includes standards addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Note strikethroughs and recommendations in the Important Considerations section for more information.

NC.K.CC.6
Identify whether the number of objects, within 10, in one group is greater than, less than, or equal to the number of objects in another group, by using matching and counting strategies.

NC.K.OA.1
Represent addition and subtraction, within 10:

- Use a variety of representations such as objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, or expressions.
- Demonstrate understanding of addition and subtraction by making connections among representations.

NC.K.OA.2
Solve addition and subtraction word problems, within 10, using objects or drawings to represent the problem, when solving:

- Add to/Take From-Result Unknown
- Put Together/ Take Apart (Result Unknown and Two Addends Unknown)

NC.K.OA.3
Decompose numbers less than or equal to 10 into pairs in more than one way using objects or drawings, and record each decomposition by a drawing or expression.

NC.K.OA.4
For any number from 0 to 10, find the number that makes 10 when added to the given number using objects or drawings, and record the answer with a drawing or expression.

NC.K.OA.6
Recognize and combine groups with totals up to 5 (conceptual subitizing).

Mathematical Practices:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?

The mathematical discourse established in Cluster 1 should continue to be embedded and utilized throughout each successive cluster.

Students determine the amount in a set either by counting, counting on/counting back, subitizing, or using other number relationships.

- Students, as they are ready, ***may*** begin to use symbols to represent conceptual language such as the = sign to represent "the same amount" as they construct equations, but they should not be required to do so. Children use number balances, bucket balances, and ten frames to explore these relationships.
- Students will likely use their fingers to keep track of addends, so it is beneficial for students to develop rapid visual recognition of the quantities 0-5 on their fingers. Teachers are encouraged to use addition and subtraction equations to model the situations. Students may begin to use equations but should not be required to.

Students continue to explore part-part-whole relationships to see that we can solve joining and separating word problems.

- Using the ability to determine an amount in a set (result unknown), students begin to explore the concept of symbols to represent this information. Children may use these symbols when explaining part-part-whole combinations of 3-10 and solve joining and separating problems. This builds from the work students began in Cluster 5 with composing and decomposing quantities of 5 and 10 as anchor numbers.
- Students develop an understanding of combining sets (result unknown) and represent addition with objects while creating a deeper understanding of the relationship between number and quantities (cardinality). This builds upon students' understanding that each successive number name refers to a quantity that is one greater in value than the previous one (inclusion). Students begin by modeling addition situations using concrete models and counting strategies to make sense of adding to and putting together.
- Students use numerous manipulatives (number balances, ten frames, rekenreks, two-color counters, connecting cubes, etc.) to continue to explore breaking a quantity into parts and examining whether the total amount is still the same (equal). For example: 5 is the same as 4 and 1. 5 is also the same as 3 and 2. 2 and 2 and 1 is the same as 5! The realization that a quantity, such as 5, can be broken apart and still be a total of 5 is the goal at this point of the year.

Important Considerations:

- Building from Cluster 5, students will continue to explore part-part-whole relationships to see that we can solve joining and separating word problems. Students' work in this Cluster making connections between composing/decomposing 10 and solving story problems will lay the foundation for their work with teen numbers in Cluster 7.
- Composing 10 and understanding addition and subtraction within 10 are combined here for students to make connections between making 10 and solving story problems.
- The concept of equality is critical in understanding part-part-whole relationships. The idea that 5 is the same as (or equal to) 3 and 2, 4 and 1, 5 and 0, 1 and 1 and 1 and 1 and 1, etc. helps children realize that numbers can be broken apart in a variety of ways and still stay the same amount.
- With the introduction of symbols, children build an understanding that the = sign does not mean, "an answer comes after me", but rather that there is a relationship between the left side of the equal sign and the right side of the equal sign. The + symbol is introduced as the "and" and the = sign is introduced as "the same as." Students represent part-part-wholes with drawings and labeling the parts with numerals. They might circle the numeral that represents the whole or put it at the top of the page. They also write using the vocabulary of number relationships (ex. 3 and 2

is 5; 5 is 4 and 1). When students are using this vocabulary freely, symbols can be introduced as the way mathematicians record the relationships. Students are expected to use symbols by the middle of the year in first grade, but some may be ready at the end of kindergarten to begin attaching the symbols with meaning.

- Geometry, measurement, and data can serve as the basis for joining and separating problems. (ex. When comparing lengths of yarn, students will now be able to say, 'how many cubes longer?' or 'how many cubes shorter?' When comparing shapes, students will now be able to state, 'how many more sides on a square than a triangle?')

Cluster 7: Foundations of Place Value - Exploring numbers 11-20

***Duration**:* 3 - 5 weeks

Content Standards:
This list includes standards addressed in this cluster, but not necessarily mastered, since all standards are benchmarks for the end of the year. Note strikethroughs and recommendations in the Important Considerations section for more information.

NC.K.NBT.1
Compose and decompose numbers from 11 to 19 into ten ones and some further ones by:

- Using objects or drawings
- Recording each composition or decomposition by a drawing OR expression
- Understanding that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

NC.K.OA.3
Decompose numbers less than or equal to 10 into pairs in more than one way using objects or drawings, and record each decomposition by a drawing or expression.

NC.K.OA.4
For any number from 0 to 10, find the number that makes 10 when added to the given number using objects or drawings, and record the answer with a drawing or expression.

NC.K.OA.5
Demonstrate fluency with addition and subtraction within 5.

Mathematical Practices:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

What is the mathematics?
The mathematical discourse established in Cluster 1 should continue to be embedded and utilized throughout each successive cluster.

Students build a foundational understanding of the base-ten number system by creating teen numbers composed of ten ones and some more ones. This work extends and formalizes all their experiences composing, decomposing, and naming quantities throughout the year. Through various experiences students have developed a deep understanding of the relationships that exist between and among numbers 0-5. By the end of the year students will be able to accurately, efficiently, and flexibly compose and decompose numbers 0-5 with automaticity.

- Students use objects, drawings, and equations to first explore and then explain how teen numbers, 11-19, are made up of ten ones and some more ones. As students gain experience in constructing and describing teen numbers in Kindergarten, they begin to move from thinking of 10 as '10 ones' to a new unit called '1 ten' (unitizing), a concept that will be formalized in Grade 1. (ex. 'I know it's 14 because there are 10 ones and 4 more ones' later can be explained as 'I know it's 14 because there is one group of 10 and 4 left over. That's 14!')

- Teachers are encouraged to use addition and subtraction equations to model the situations, but students are not required to use equations until Grade 1.

Fluency to 5

- At this point in the learning sequence, students should be able to quickly recognize small groups of numbers (0-5) in a variety of arrangements (perceptual subitizing). We now need to move forward from perceptual subitizing to conceptual subitizing in helping students to understand that smaller groups of numbers can be joined to make a larger number.

Building on students opportunities to explore, represent, and discuss various ways to compose and decompose quantities 0-5, students have developed a deep understanding of the relationships that exist between and among numbers. Students are now able to accurately, efficiently, and flexibly compose and decompose numbers 0-5 with automaticity. While students have been working on 5 most of the year, it is included in this cluster because this is the point that mastery is expected.

Important Considerations:

- The number range in this standard emphasizes the understanding of ten ones rather than an understanding of the tens "place". This work is an opportunity for students to begin using the strategy of counting on.
- Working with numbers 11 - 19 builds foundational understanding of place value. It is important for students to understand that 18 is made up of a group of 10 ones and 8 more ones instead of just 18 individual ones.
- Students will need varied and multiple experiences in order to ensure understanding of a set of 10 ones and some more ones when working with numbers 11 - 19.
- Special attention should be paid to the number names 11-19. This is the first opportunity students have to work with 2-digit numerals, yet they do not follow the same pattern of naming as those numerals 20-99.
- It is important that students understand the word "grouping." This term is used frequently when working with a set of objects. This word can become problematic if not fully understood.
- Even though students may have demonstrated fluency, they still need to be able to apply this knowledge as needed in addition and subtraction problem situations.