Formative Assessment: Bakers Gonna Bake	
Frameworks Cluster	Division of Fractions Conceptions Cluster
Standard(s)	**NC.6.NS.1** *Use visual models and common denominators to:* ● *Interpret and compute quotients of fractions* ● *Solve real-world and mathematical problems involving division of fractions* **SMP 1** *Make sense of problems and persevere in solving them* **SMP 4** *Model with mathematics* **SMP 5** *Attend to precision* **SMP 6** *Use appropriate tools strategically* **SMP 7** *Look for and make use of structure* **SMP 8** *Look for and express regularity in repeated reasoning*
Materials/Links	● Recording sheet (can be given to each individual student or can be displayed for the students to answer on their own) ● Fraction Tiles
Learning Goal(s)	Students will be able to solve a multi-step division of fraction problem involving a whole number and a non-unit fraction.

Task Overview:
This formative assessment task provides the teacher an opportunity to determine student understandings about dividing fractions.

Prior to Task:
- Teach modeling division of fractions.
- Be sure students have access to paper and fraction tiles for the activity. Students are not required to use these materials, but the tools should be made available to them.

Teaching Notes:
Directions:
- Pose the question to your students. You may use the following recording sheet for students to show their thinking, however, you could also project the question and have them do their work on a separate sheet of paper.

Use this formative assessment after the Zane's Zoo Adventure task and/or after students have had time to practice dividing whole numbers by non-unit fractions.

Student sheets begin on next page.

Name ____ Date ________________________

Bakers Gonna Bake

Student Recording Sheet

Part 1: To make 5 dozen cookies, Martin needs ⅝ cup of brown sugar.
Martin has 2 ¾ cups of brown sugar. How many cookies can Martin make? Model your answer.

Part 2: If Martin only wants to make 1 dozen cookies, how much brown sugar does he need? Model your answer.

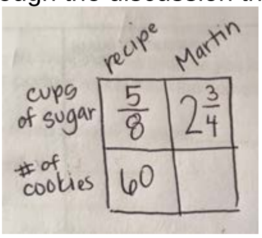
Work Area:

Possible Strategies/Anticipated Responses:

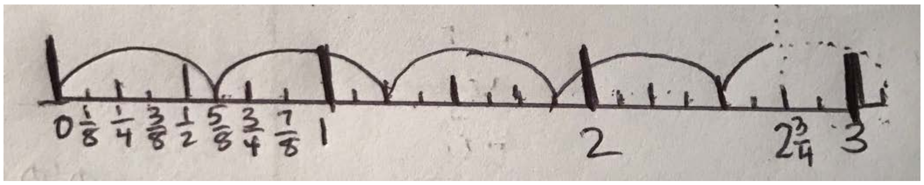
- Consider using the Class Discussion Planner**:** https://tinyurl.com/discussion-planner

Part 1:

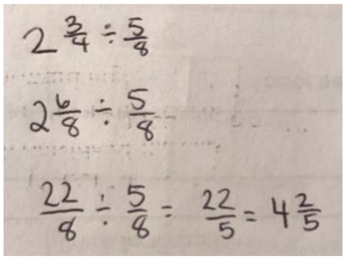
- Some students may express their answer as 22 dozen, while others may say 264 cookies.
- Some students may know to divide and get to the point where they determine that $2\frac{3}{4} \div \frac{5}{8}$ is $4\frac{2}{5}$ and then get stuck. If this happens help with using labels to determine that this means that Martin can make $4\frac{2}{5}$ batches. After determining that they will be able to multiply by 60 to determine that Martin can make 264 cookies.
- Some students may connect this problem to the use of proportional relationships in the previous cluster. By incorporating the use of a ratio box students can see that by dividing $2\frac{3}{4}$ by ⅝ you can determine the multiplicative relationship within the cups of brown sugar. Further, they can determine the relationship between the cups of brown sugar and cookies by dividing 60 by ⅝. By organizing the order in which students share their work, students may have a better understanding of this task, through the discussion that connects these different strategies.



- Some students may realize that because it requires ⅝ of a cup of brown sugar to make 5 dozen cookies it takes ⅛ of a cup to make one dozen cookies. Since there are 22 eighths in $2\frac{3}{4}$ Martin can make 22 dozen cookies, or 264 cookies.
- Students may use a number line for their division:

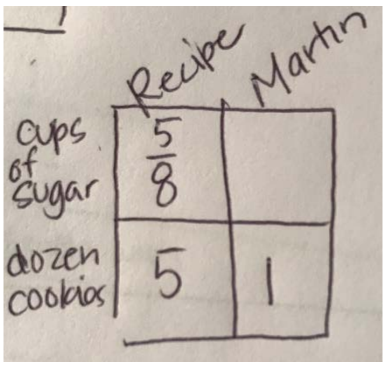


- Students may use finding a common denominator and then using an improper fraction to determine the quotient. In using this strategy, they may struggle with the order of work they should complete; common denominator then improper fraction vs. improper fraction and then common denominator. Either way works and this can be investigated in your conversation.



Part 2

- If using the ratio box in part 1, you may want to have students return to it for part two. By doing this they might be able to see the "friendly" multiplicative relationship between a dozen of cookies and an eighth of a cup, and then easily divide ⅝ by 5 to determine that Martin would need ⅛ of a cup of brown sugar to make his single dozen of cookies.



Making Bows	
Frameworks Cluster	Division of Fractions Conceptions Cluster
Standard(s)	**NC.6.NS.1** *Use visual models and common denominators to:* ● *Interpret and compute quotients of fractions* ● *Solve real-world and mathematical problems involving division of fractions* **SMP 1** *Make sense of problems and persevere in solving them* **SMP 4** *Model with mathematics* **SMP 5** *Attend to precision*
Materials/Links	● Recording sheet (can be given to each individual student or can be displayed for the students to answer on their own) ● Fraction Tiles
Learning Goal(s)	Students will be able to solve a division of fraction problem involving a whole number and a non-unit fraction.

Task Overview:
This formative assessment task provides the teacher an opportunity to determine student understandings about dividing fractions.

Prior to Task:
Teach modeling division of fractions.

Be sure students have access to paper and fraction tiles for the activity. Students are not required to use these materials, but the tools should be made available.

Teaching Notes:
Directions:

- Pose the question to your students. You may use the following recording sheet for students to show their thinking, however, you could also project the question and have them do their work on a separate sheet of paper.

Student sheets begin on next page.

Name ____ Date _________________________

Making Bows

Student Recording Sheet

Part 1: Keyla is making bows for her school's dance team. She needs ⅞ continuous yards of ribbon for each bow. How many bows can she make from a 6-yard roll of ribbon? Model your answer.

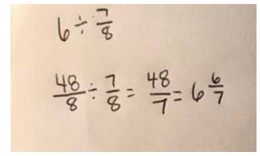
Part 2: Keyla wants to make 1 bow for each of the 20 girls on the dance team. She knows that she needs 17 ½ yards of ribbon to do this. Keyla believes that 3 6-yard rolls of ribbon will be enough, but her mom insists that she needs 4 rolls. Who do you agree with? Justify your choice with an illustration.

Work Area:

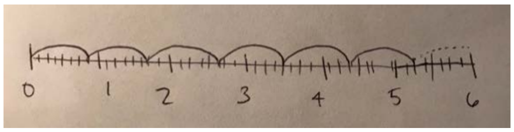
Possible Strategies/Anticipated Responses:

Part 1: Answer: 6 bows

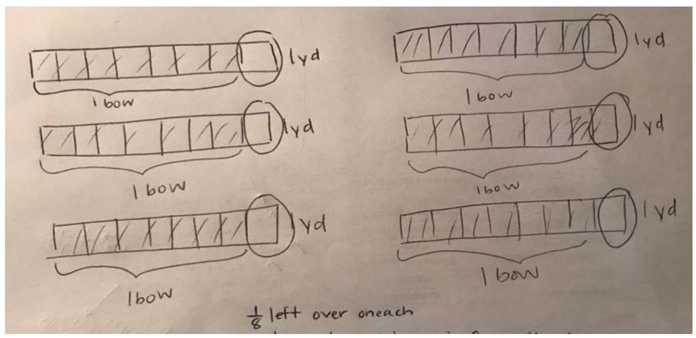
- Students may use, but are not limited to, the following strategies:
  - Students may use the standard algorithm with common denominators.



  - Students may use a number line to determine that Keyla could make six full bows but not have enough left to make a 7th bow.



  - Students may use logical reasoning to determine that there could be one bow made from each yard (for a total of six bows) and that there would be ⅛ of a yard left from each yard. Since there are only three eights left on each roll that would not be enough to create another bow.



  - If students are struggling, you may want to have fraction strips made out of 8ths that students can manipulate and determine how many sets of ⅞ they can make.
- Part 2: Answer: Keyla's mom is correct.
  - This part of the question is intended to address making sense of division problems. Keyla is partially correct because she has enough *yards* of ribbon to make the bows, but two of the bows would have to be made with ribbon from multiple rolls, which wouldn't be an appropriate way to make a bow.
- After students have determined the number of rolls and created an argument for agreeing with Keyla's mom, summarize the lesson through a whole-group discussion of strategies used for these two tasks. Consider using the [Class Discussion Planner](https://tinyurl.com/discussion-planner): https://tinyurl.com/discussion-planner

  to guide your discussion.

Making Sense of Models	
Frameworks Cluster	Division of Fractions Conceptions Cluster
Standard(s)	**NC.6.NS.1** *Use visual models and common denominators to:* ● *Interpret and compute quotients of fractions* ● *Solve real-world and mathematical problems involving division of fractions* **SMP 1** *Make sense of problems and persevere in solving them* **SMP 4** *Model with mathematics*
Materials/Links	● Recording sheet (can be given to each individual student or can be displayed for the students to answer on their own paper) ● Fraction tiles and fraction pictures
Learning Goal(s)	Students will be able to compare division of fraction models by interpreting models to create a numerical equation involving division.

Task Overview:

This formative assessment task provides the teacher an opportunity to determine student understandings about modeling division of fractions.

Prior to Task:

Make sure to use prior instructional time to model division of fractions with fraction tiles and various pictures that might model division of fractions conceptually in various ways.

Teaching Notes:

Directions:

- Pose the question to your students. You may use the following recording sheet for students to show their thinking, however, you could also project the question and have them do their work on a separate sheet of paper.

Use this formative assessment after the *Fraction Division Sort* task and after students have had time to work and practice with models.

Student sheets begin on next page.

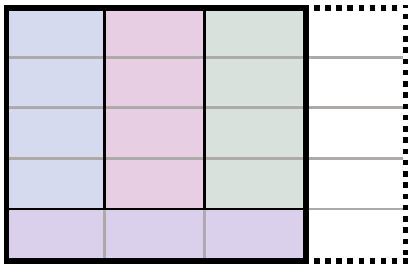
Name ____ Date _________________________

Making Sense of Models

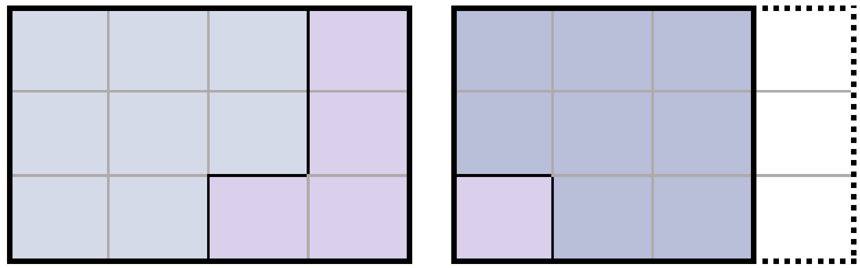
Student Recording Sheet

Part 1: Write a numerical equation for each of the models. Explain how you determined the equation for one of the models.

Model 1



Model 2



Numerical Equations:

Model 1: ______________________________

Model 2: ______________________________

Model for explanation (circle one): Model 1 Model 2

Explanation:

Part 2: Using the other model, write a word problem that you could use the model to solve.

Model for word problem (circle one): Model 1 Model 2

Word Problem:

Possible Strategies/Anticipated Responses:

- Model 1:
    - Expression: $\frac{3}{4} \div \frac{1}{5}$
    - Students may initially think that this model is $\frac{3}{4} \div 4$ because it is divided into four parts, however the four parts are not equal therefore the students will have to look more closely.
    - The explanation can include things like that the three equal parts each have 4 out of 20 parts which would be equivalent to one fifth (the 4 rectangle pieces equivalent to one fifth) and that the smaller piece has 3 out of the four parts needed so the quotient would be $3\frac{3}{4}$
    - When writing a word problem, students may mistakenly write about dividing by five. (One example: *There was ¾ of a container of hot chocolate mix left over from the party. If it takes ⅕ of the container to make a cup of hot chocolate, how many cups of hot chocolate could be made?*)
- Model 2:
    - Expression: $1\frac{3}{4} \div \frac{2}{3}$
    - Students may initially think that this model is $1\frac{3}{4} \div 3$ because it is divided into three parts, however the three parts are not equal, therefore the students will have to look more closely.
    - The explanation can include things like that the two equal parts each have 8 out of 12 parts, which is equivalent to ⅔, and that the smaller piece has 5 out of the 8 parts needed to complete a two thirds fraction, so the quotient would be $2\frac{5}{8}$.
    - When writing a word problem, seeing the attempted grouping into ⅔ is key. (*Dad has 1 ¾ cups of flour. The mini-muffins recipe calls for ⅔ of a cup of flour. How many batches of mini-muffins could he bake?*)
- Consider using the Class Discussion Planner. https://tinyurl.com/discussion-planner

Formative Assessment-Running Laps	
Frameworks Cluster	Division of Fractions Conceptions Cluster
Standard(s)	**NC.6.NS.1** *Use visual models and common denominators to:* ● *Interpret and compute quotients of fractions* ● *Solve real-world and mathematical problems involving division of fractions* **SMP 1** *Make sense of problems and persevere in solving them* **SMP 4** *Model with mathematics* **SMP 5** *Attend to precision* **SMP 6** *Use appropriate tools strategically* **SMP 7** *Look for and make use of structure* **SMP 8** *Look for and express regularity in repeated reasoning*
Materials/Links	● Recording sheet (can be given to each individual student or can be displayed for the students to answer on their own) ● Fraction Tiles
Learning Goal(s)	Students will be able to solve a multi-step division of fraction problem involving time and rates.

Task Overview:
This formative assessment task provides the teacher an opportunity to determine student understandings about dividing fractions and applying rate concepts.

Prior to Task:
Teach modeling division of fractions.
Be sure students have access to paper and fraction tiles for the activity. Students are not required to use these materials.

Teaching Notes:
Directions:
- Pose the question to your students. You may use the following recording sheet for students to show their thinking, however, you could also project the question and have them do their work on a separate sheet of paper.

Student sheets begin on next page.

Name ________________________________ Date ____________________

Running Laps

Student Recording Sheet

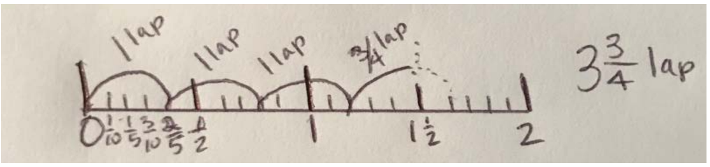
Part 1: Keri runs about $1\frac{1}{2}$ mile in 15 minutes. If the track she runs on is $\frac{2}{5}$ mile around, how many laps will she run in the following amounts of time: 15 min, 30 min, 45 min 1 hour?
Draw an illustration or model to justify your answer.

Part 2: Keri wants to run 10 km (approximately 6.2 miles).
If she is running on a track that is $\frac{2}{5}$ of a mile around, how many laps should Keri run?

Work Area:

Possible Strategies/Anticipated Responses:

- Part 1: 15 minutes: $3\frac{3}{4}$ laps, 30 minutes: $7\frac{1}{2}$ laps, 45 minutes: $11\frac{1}{4}$, 60 minutes 15 laps
  - Students may use a number line to determine the number of laps in $1\frac{1}{2}$ miles, which is $3\frac{3}{4}$.



  - Students may use fraction strips to show that Keri would run three complete ⅖ mile laps and that the fourth lap would have to be incomplete. By using tenths they can show that she would need to complete three of the four parts of the fourth lap for a total of $3\frac{3}{4}$ laps.

  - Some strategies students may use to determine the number of laps run in the number of minutes beyond the initial 15:
    - Students may continue to rely on their fraction models (either number line or fraction strips) to extend into double, triple, and quadruple the time.
    - Students may use repeated addition to determine the number of laps (i.e. $3\frac{3}{4}+3\frac{3}{4}=7\frac{1}{2}, 7\frac{1}{2}+3\frac{3}{4}=11\frac{1}{4}, etc$).
    - Students may use multiplication to determine the number of laps (i.e. $3\frac{3}{4}\cdot 2, 3\frac{3}{4}\cdot 3, etc$).
    - Students may use division to determine the number of laps (i.e. in thirty minutes she runs 3 miles, so $3 \div \frac{2}{5} = 7\frac{1}{2}$).
- Part 2: Keri would need to run $15\frac{1}{2}$ laps.
  - Students should use some reasoning from the information they gained in part 1 to determine that the number of laps will have to be greater than 15 because they found that in the 60 minutes she ran 15 laps which equates to six miles. They can use this knowledge to move forward to just determine the number of laps needed to run $\frac{2}{10}$ of a mile and add that to 15.
  - Students may be confused about whether to use the 10 from 10 km or the 6.2 for miles. Encourage them to use the 6.2 because the laps are given in miles.
  - Students will need to change 6.2 to $6\frac{2}{10}$. This should be an easy conversion because 6.2 in words is “six and two tenths” which is directly related to the fraction form. If a student is struggling with it consider having them talk it out and get to this same conclusion.
- After students have determined their answers for part one and part two, as well as created justification for their answers, summarize the task through a whole-group discussion of strategies used for determining their solutions. Consider using the [Class Discussion Planner](https://tinyurl.com/discussion-planner): https://tinyurl.com/discussion-planner

Defending Denominators	
Frameworks Cluster	Reasoning with Factors and Multiples
Standard(s)	**NC.6.NS.4** *Understand and use prime factorization and the relationship between factors to:* ● *Find the unique prime factorization for a whole number.* ● *Find the greatest common factor of two whole numbers less than or equal to 100.* ● *Use the greatest common factor and the distributive property to rewrite the sum of two whole numbers, each less than or equal to 100.* ● *Find the least common multiple of two whole numbers less than or equal to 12 to add and subtract fractions with unlike denominators.* **SMP 2** *Construct viable arguments and critique the reasoning of others.* **SMP 4** *Model with mathematics.* **SMP 7** *Look for and make use of structure.*
Materials/Links	Print one recording sheet per student, or project the problem for students to see (and have paper and pencils available for them to show their work).
Learning Goal	Students will demonstrate their understanding of finding a least common multiple (LCM) using their choice of methods.

Task Overview:
This formative assessment evaluates students' understanding of how to determine the least common multiple (LCM) for the purpose of finding a common denominator.

Prior to Lesson:
- Students should have found Least Common Multiples using at least two different methods (prime factorization, listing multiples). Allow students to use both methods while choosing a method they prefer. Later you can discuss what specific situation(s) might make one method more efficient or beneficial than the other.
- Students should be able to apply their knowledge of LCM to adding and subtracting fractions with unlike denominators.
- Help students understand that ANY multiple will work for finding common denominators; however the LCM is the most efficient.

Teaching Notes:

- The purpose of this assessment is to have students generalize that using the LCM will simplify the process of adding (or subtracting) fractions.

Directions:
1. Pose the question at the top of the student handout to your students. You may use the recording sheet for students to show their thinking, however, you could also project the question and have them do their work on a separate sheet of paper.
2. After students have come up with their defense (#3), summarize the possibilities through a whole-group discussion of the question, focusing on the students' logic as well as their calculations. Consider using the Class Discussion Planner (https://tinyurl.com/discussion-planner) to guide your discussion.
3. If no students used a number line in their solution or justification, use a number line to model finding the sum, helping students make connections between finding common units on the number line (so that the fractions will be able to be combined) and finding the common denominator when adding using the fraction.

This formative assessment is designed to be used after teaching about using common denominators for adding and subtracting fractions with denominators up to 12, and could be used after using the *Bake Off!"*and *Two Way Street* lessons.

Lesson plan template adapted from *Taking Action: Implementing Effective Mathematics Teaching Practices*, NCTM, 2017

Student sheets begin on next page.

Name ____ Date ________________________

Defending Denominators

Mr. Smith asked his class to add $\frac{3}{8} + \frac{3}{12}$ and simplify the answer.

Brody plans to create equivalent fractions using 48 as the denominator before adding.

a.) Will his method work? Explain why or why not.

b.) What is a more efficient way to solve this problem?

c.) Use the method you described in part b to find the sum.

Possible Strategies/Anticipated Responses:

A. Explanations will vary. Brody's method will work, because 48 is a multiple of both denominators, and *any* multiple of both denominators can be used to create fractions that are equivalent to the original fractions. If equivalent fractions are calculated correctly, the new expression will be $\frac{18}{48} + \frac{12}{48} = \frac{30}{48}\ or\ \frac{5}{8}$

B. Explanations will vary. A more efficient way to solve the problem is to use 24 because it is the least common denominator. This is helpful because using the LCM as a common denominator usually requires little or no work to simplify the fraction, because the calculations involve smaller numbers. Also, since the numbers in the numerator and denominator are smaller, they are easier to understand and work with.

C. If the LCM is used as the common denominator, the expression becomes $\frac{9}{24} + \frac{6}{24} = \frac{15}{24}\ or\ \frac{5}{8}$

Watch for these common errors:

- Some students may still be adding the numerator *and the denominator*, resulting in the sum of $\frac{6}{20}$.
    - Suggestions for remediating these students:
        - You may need to return to adding fractions with like denominators and discuss how the denominator is similar to a label: 1 fourth + 1 fourth is 2 fourths, just like 2 cats + 4 cats is 6 cats, or 3 $5 bills + 4 $5 bills is 7 $5 bills. The same is true for fractions; you have to have a common denominator to add.
        - Continuing to allow these students to model with fraction tiles may be helpful, so the students can visually see the connection between equivalent fractions with different denominators and have a physical reminder that if you put fractions with two different denominators together, there is no way to name the resulting mismatched collection of fractions (or fraction tiles)
        - Consider revisiting the Fraction Frenzy task.
- Some students may say that using 9 6as the denominator is the most efficient, possibly because 96 is the product of the two denominators (8 and 12), or they might not realize that there was a common multiple lower than 96.
    - Suggestions for working with these students
        - Be sure that students understand that using 96 or any other common multiple besides the LCM is not *wrong*, it just ends up being more work most of the time, and is often more prone to error.
        - If students need convincing, you could have them write five equivalent fractions for $\frac{3}{8}$ and five equivalent fractions for $\frac{3}{12}$ to see if they notice any other equivalent fractions with common denominators. Because there are smaller multiples of 8 and 12 that could be used, help students see that this makes the work much more efficient and provides a final outcome that is in numbers that are easier to understand work with.
- Some students may say that using 48 or 72 as the denominator is the most efficient because these are also common multiples of the two denominators (8 and 12). These students understand the purpose of finding a common denominator but aren't recognizing all of the multiples for one or both of the denominators.
    - Suggestions for working with these students:
        - If students need convincing that their denominator is not the <u>Least</u> Common Multiple, you could have them write five equivalent fractions for $\frac{3}{8}$ and five equivalent fractions for $\frac{3}{12}$ to see if they notice any other equivalent fractions with common denominators. Because there are smaller multiples of 8 and 12 that could be used, help students see that this makes the work much more efficient and provides a final outcome that is in numbers that are easier to understand work with.