Growing Factor Trees

Frameworks Cluster	Reasoning with Factors and Multiples
Standard(s)	**NC.6.NS.4** *Understand and use prime factorization and the relationship between factors to:* ● *Find the unique prime factorization for a whole number.* ● ~~*Find the greatest common factor of two whole numbers less than or equal to 100.*~~ ● ~~*Use the greatest common factor and the distributive property to rewrite the sum of two whole numbers, each less than or equal to 100.*~~ ● ~~*Find the least common multiple of two whole numbers less than or equal to 12 to add and subtract fractions with unlike denominators.*~~ **SMP 1** *Make sense of problems and persevere in solving them.* **SMP 7** *Look for and make use of structure.* **SMP 8** *Look for and express regularity in repeated reasoning.*
Materials/Links	Optional: One copy of the student recording sheet per student, copied 2-sided (this task can be done on lined paper if the questions are projected for students.)
Learning Goals	● Students will understand that when finding a number's prime factorization, there are multiple ways to begin a factor tree ● Students will become more fluent in finding the prime factorization of a number, using factor pairs to create a factor tree

Task Overview:

Students will work with a partner to create factor trees starting with any factor pairs.

Prior to Lesson:

- Plan partner assignments in advance.
- In fourth grade students, found factor pairs for whole numbers up to and including 50. They also developed an understanding of prime and composite numbers. *The part of NC.6.NS.4* addressed in this lesson builds upon that knowledge and *extends to whole numbers up to and including 100.*
- Prior to this activity, students should be taught how to create a factor tree, beginning with any two factors (not including 1), and rewriting the prime numbers in expanded form, in order from least to greatest (you may choose to then rewrite using exponents, but that is optional for this lesson).
- Review vocabulary: *factors, factor pairs, product, prime, and composite*.

Teaching Notes:

Task launch:

- Play a factor pair race game where students list as many factor pairs of a number that they can think of in 30 seconds. The time and/or number selected can be altered to adjust the challenge level. Choose several from this list of suggested numbers: 12, 18, 22, 24, 30, 32, 36, 42, 45, 48, 50, 60, and 100.
- Allow students who are not yet fluent with multiplication facts to use a multiplication chart. You may need to show them how to search for their number (product) within the chart to find the factors. Scaffolding the task in this way allows students to focus on grade-level content despite their lack of fact fluency.

Directions:

1. Have students individually answer the first question on the student handout, making a prediction about whether the prime factorization of a number will always be the same regardless of the factor pair chosen for the first step in the factor tree. When students are finished answering, consider polling students on their predictions (yes or no) for follow-up later, but do not discuss reasoning yet.
2. Working with a partner, students will list all of the factor pairs of 24, the first number in the set of numbers listed in question 2 of the handout. Students can use a multiplication chart or the Factorize Activity from Illuminations for support on this step if needed. https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Factorize/.

3. When all groups have finished listing the factor pairs, ask all groups to share the number of factor pairs they found. They should have found 4 pairs (1x24, 2x12, 3x8, 4x6), so if some groups listed fewer or more pairs, this is an opportunity to discuss common errors, such as *not including 1x___* as a factor pair; r*epeating factor pairs in a different order*, e.g. 3x8 and 8x3; or *not including all factor pairs* (which often happens when students do not list factor pairs in an organized way).
4. After ensuring that all groups now have the 4 correct factor pairs listed, direct students to each choose a different factor pair than their partner, then create a factor tree for 24 using their chosen factor pair. When both partners are finished, they should discuss their factor trees, identifying similarities and differences.
5. Briefly discuss the similarities and differences found. After each time a student shares a similarity or difference, ask other groups if the observation matches what they noticed in their own work. In this way, students can catch and fix any errors as a natural part of the discussion. *Do not allow the discussion to extend to generalizing for all starting numbers yet.*
6. Instruct students to continue this process for 36 and 40, the other two numbers listed in question 2, stopping to see if the similarities and differences they found with 24 extend to these other numbers. Then they should discuss and answer questions 3 and 4 on the handout.
7. For early finishing groups, extend the activity using Possible Extension questions below.
8. When all groups have finished, ask them to revisit their prediction from the beginning of the task. Poll the class on the same question (Will the prime factorization of a number will always be the same regardless of the factor pair chosen for the first step in the factor tree?). Follow up by asking if anyone changed their answer, and why.
9. Summarize the lesson through a whole-group discussion of prime factorization, focusing on similarities, differences, and patterns students observed, and generalizations they made, either (1) within their two factor trees for the same number, or (2) between the factor trees for different numbers. Consider listing students' observations on chart paper (without indicating whether an observation is correct or not) to help guide the discussion. The Class Discussion Planner (https://tinyurl.com/discussion-planner) may also be used as a guide.
    - Some patterns students may find:
        - Regardless of the starting number, 1x___ is always a factor pair, but it is not helpful when creating a factor tree.
        - Even numbers have a factor of two
        - Numbers that end in 0 or 5 have five as a factor
        - Numbers whose digits sum to three have three as a factor
        - Regardless of the factor pair they start with, the prime factorization remains the same
    - Questions you may ask to probe student thinking:
        - How do you determine which factor pair you would like to start with?
        - What would happen if you start with the same factor pair but choose a different pair in your second step?
        - As a class, have we found all of the ways that a factor tree for (number) can be created?

Possible Extensions:

- Allow groups to complete more than 2 factor trees and/or allow students to work on more difficult numbers. Consider using 60, 64, 72, 84, or 90.
- Challenge students to find two numbers that have *almost* the same prime factorization -- the prime factorization being the same except for one prime factor.
- Challenge students to find two numbers that have *completely different* prime factorizations -- the prime factorizations not having any prime factors in common.

This is the first in a series of factor/multiple tasks. It can be followed by the "How Many Ways?" task.

Lesson plan template adapted from *Taking Action: Implementing Effective Mathematics Teaching Practices*, NCTM, 2017

Student sheets begin on next page.

Name _____ Date _____

Growing Factor Trees

1. Make a prediction: Will a number's prime factorization always be the same, regardless of which factor pair you start with? Explain.

2. On the back of this paper, list all factor pairs for the number 24. Each partner should choose one of the factor pairs to start the factor tree for that number and use the factor trees to determine the prime factorization. Record your work on the back of this paper. When your teacher tells you it is time, follow the same process for the numbers 36 and 40.

Answer questions 3 & 4 after completing the back of this paper.

3. Does the factor pair you chose to start with impact the prime factorization of a number? Justify your answer.

4. How does the factor pair you choose impact the ***process*** for finding the prime factorization of a number?

24		
Factor pairs:	Factor tree 1 Prime factorization:	Factor tree 2 Prime factorization:

36		
Factor pairs:	Factor tree 1 Prime factorization:	Factor tree 2 Prime factorization:

40		
Factor pairs:	Factor tree 1 Prime factorization:	Factor tree 2 Prime factorization:

Possible Strategies/Anticipated Responses:

Factor Pairs:

24: 1 and 24, 2 and 12, 3 and 8, 4 and 6
36: 1 and 36, 2 and 18, 3 and 12, 4 and 9, 6 and 6
40: 1 and 40, 2 and 20, , 4 and 10, 5 and 8

Prime Factorization*:

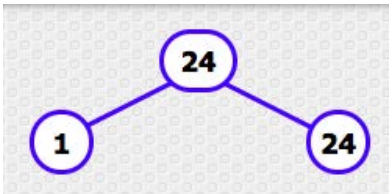
- $24: 2 \cdot 2 \cdot 2 \cdot 3$ *or* $2^3 \cdot 3$
- $36: 2 \cdot 2 \cdot 3 \cdot 3$ *or* $2^2 \cdot 3^2$
- $40: 2 \cdot 2 \cdot 2 \cdot 5$ *or* $2^3 \cdot 5$

*Students may write the prime factorization in expanded form if they have not learned/mastered exponents yet.

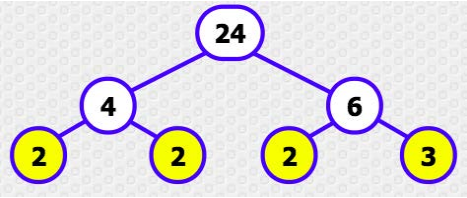
For question 3: The factor pair you start with does not impact the prime factorization of the number.

For question 4: Answers will vary (and could depend on student preference), but may include:

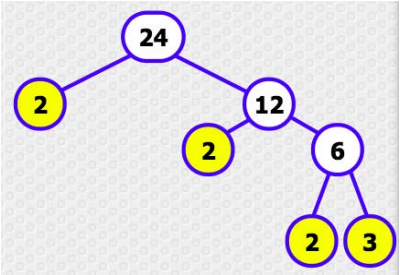
- It's "easiest" to start with 2x____ if the number is even, because it's not hard to find half of a number.
- It's "easiest" to start with (number) because I know that numbers multiplication facts best.
- Starting with 1x___ doesn't help because you still have the same number you started with.



- Choosing the numbers that are closest together to start with will typically make shorter work (fewer rows) of finding the prime factorization.



- The factor tree only grows on one branch if you always begin with a prime factor for each factor pair



- Factor trees' branches stop growing when they get to a prime number

- Students may stop early in their finding of the prime factorization, not realizing that factors of a number can be composite, therefore stopping at a factor pair that is not two primes. If this happens you may have to do a prime/composite review.
- Even when choosing the same factor pair to start with there are multiple trees that can be created. As you start it should be celebrated that students still get to the same end result.
- Students may struggle coming up with all of the factor pairs. If this is an issue, encourage students to list

factor pairs in an organized manner (1 x ___, then checking to see if 2 x ___, 3 x ___, … are possible). Consider allowing students who are not fluent with multiplication factors to use of a multiplication chart as well.

- Students may not realize the prime factorizations are the same if they list the numbers in a different order (2x2x3 vs 2x3x2). If this happens, you may want to review commutative property of multiplication.

How Many Ways?

Frameworks Cluster	Reasoning with Factors and Multiples
Standards	**NC.6.NS.4** *Understand and use prime factorization and the relationship between factors to:* ● ~~*Find the unique prime factorization for a whole number.*~~ ● *Find the greatest common factor of two whole numbers less than or equal to 100.* ● *Use the greatest common factor and the distributive property to rewrite the sum of two whole numbers, each less than or equal to 100.* ● ~~*Find the least common multiple of two whole numbers less than or equal to 12 to add and subtract fractions with unlike denominators.*~~ **SMP 7** *Look for and make use of structure.* **SMP 8** *Look for and express regularity in repeated reasoning.*
Materials/Links	● One copy of the recording sheet per student (if you choose) ● Individual dry-erase boards and markers (one for every 2 students), if available and desired
Learning Goals	Students will be able to identify the Greatest Common Factor and rewrite the sum of two numbers using the GCF and the distributive property. They will understand that the number of common factors two numbers have impacts the ways that the sum (or difference) of two numbers can be rewritten.

Task Overview:

This learning activity encourages students to use factors to see the multiple ways that a sum can be rewritten using the distributive property.

Prior to Lesson:

- In 5th grade (NC.5.OA.2) students gained an understanding of the distributive property. The task launch is intended to refresh their memory on this topic.
- Students need to know how to find all common factors of two numbers and then determine which is the GCF.
- Plan partner assignments in advance.

Teaching Notes:

Consider allowing students who are not yet fluent with multiplication and division facts to use a multiplication table throughout this lesson. This allows the student to focus on this grade-level standard without their lack of fluency being a barrier. Note that students may need to be taught how to use a multiplication table in this way (working "backwards" to find the factors of various numbers).

Task Launch:

- Present students with the expression 6(2 + 3). Ask students to simplify the expression by applying the distributive property. (Students should have exposure to distributive property from 5th grade.) Monitor students while they simplify the expression. You may need to have a class discussion on what distribute means, and include examples using the area model.
- If students are struggling to remember what they learned in 5th grade and need more practice applying distributive property, work on building that concept using the following expressions before moving on. When the class is ready, come back to the original expression 6(2 + 3).
    - 2(5 + 3)
    - 7(6 + 11)
    - 5(10 + 7)
    - 4(12 + 7)
    - 3(8 + 5)

- Once students have had ample time to work and the majority of your students have the expression 12 + 18, ask students to talk with their shoulder buddy about ways that the numbers 12 and 18 are similar, or what they have in common. Guide the conversation toward the factors of both numbers. Some students may point out they are both divisible by 2 (or even), or divisible by 3. They may also notice that both numbers are also divisible by 6, and may realize that 6 is the number outside the parentheses in the original expression.
- Have students make a list of factor pairs for both 12 and 18, and then notice the common factors they have. Point out the greatest common factor is 6. Ask students to derive the steps of how to get 12 + 18 back to the expression 6(2 + 3). Explain to students that they factored out the greatest common factor.
- Pose the question to students, "How can you rewrite 12 + 18 with the distributive property, using the common factor of 2 or common factor of 3 instead of the greatest common factor of 6?" Have students work with their shoulder buddy to answer the question, with one student using the common factor of 2, and the other student using the common factor of 3. When finished, students can explain their expression to their partner. Follow up by selecting students to explain their work to the class.

Directions:

1. Using paper and pencil or dry-erase boards and markers, have students work in pairs to find the GCF of the pairs of numbers in the following expressions, then rewrite the sum using the GCF and the distributive property:
    a. 16 + 28
    b. 18 + 6
    c. 30 + 27
    d. 24 + 9
2. After students complete a – d, have them work with their partner to determine whether any of the sums can be rewritten using the distributive property with a factor other than the GCF. Allow students to share out their thinking and process before beginning the next step.
3. Pose the question, "How many ways can you rewrite the sum of 24 and 36 using the distributive property?" to your students. Students should work individually for this task. You may use the following recording sheet for students to show their thinking; however, you could also project the question on an interactive whiteboard or write it on a dry-erase board and have them do their work on a sheet of paper.
4. Give students ample time to come up with multiple methods for rewriting the sum of two numbers using the distributive property.
5. When students say they are finished, ask how they know they have found all possible ways?
6. After students have found the possibilities for rewriting the sum, wrap up by providing an opportunity for students to share their solutions, focusing on *different* strategies that students used to determine the common factors used to rewrite the expressions using the distributive property, as well as discussing why this number pair has so many options and why others might not. Consider using the Class Discussion Planner (https://tinyurl.com/discussion-planner) as a guide.
7. Extensions (optional):
    a. Ask students to determine whether there would be multiple ways to rewrite other sums. Some sums you could use are: 25 + 40 (one way, because the only common factor is 5), 42 +4 8 (three ways, because the common factors are 2, 3, and 6).
    b. Challenge students to find their own pair of numbers whose sum can be rewritten using the distributive property (1) only one way, (2) only two ways, (3) only 3 ways (and so on), and (4) cannot be rewritten using the distributive property (no common factors other than 1)

This lesson can be used after *Growing Factor Trees* to help develop skills in prime factorization and its application.

Lesson plan template adapted from *Taking Action: Implementing Effective Mathematics Teaching Practices*, NCTM, 2017

Student sheets begin on next page.

Name _____ Date _____

How Many Ways?

Find the common factors for 24 and 36.

24	**36**

Common Factors	

List all the ways the distributive property can be used to write expressions equivalent to 24+36, then circle the expression that uses the GCF.

Why are there so many ways to rewrite the sum of 24 and 36?

What are two numbers whose sum *cannot* be rewritten using the distributive property?

Possible Strategies/Anticipated Responses:

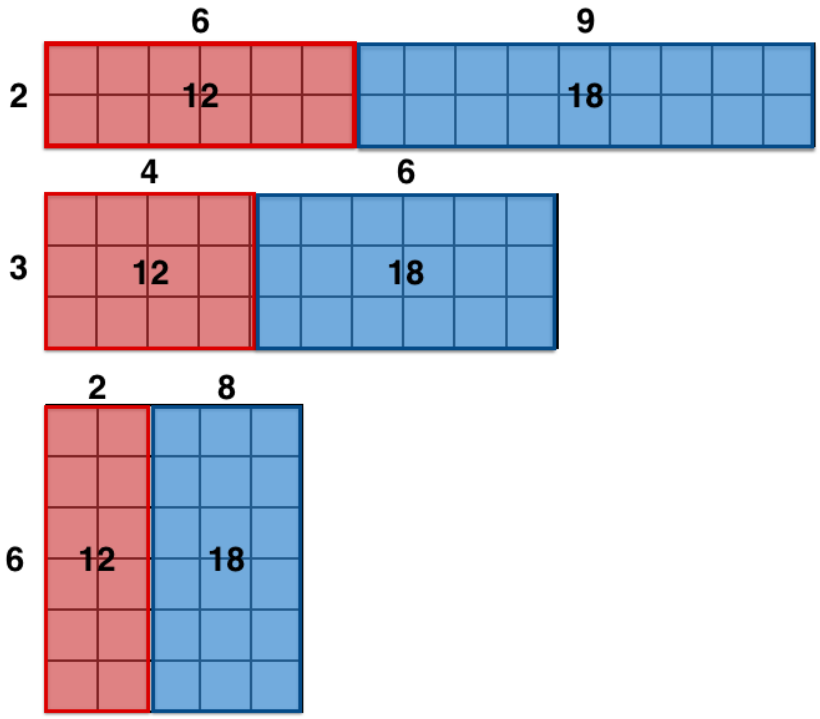
The ways that students can rewrite the sum are:

- 2(12 + 18)
- 3(8 + 12)
- 4(6 + 9)
- 6(4 + 6)
- 12(2 + 3) – which is the expression that should be circled because it uses the GCF of 24 and 36, 12.

There are multiple ways to rewrite the sum of 24 and 36 because they share many common factors. Any pair of addends that do not share a common factor (other than 1) cannot be rewritten using the distributive property. Addends that share only one common factor (besides 1) can be rewritten in just one way.

A common error is finding *some* but not *all* factors of a number, which can lead to students believing they have rewritten a sum using the GCF when they have actually used a different common factor. Encourage these students to list their factor pairs in order, starting at 1 x ___, continuing number by number until they get to a factor pair that is "doubles" (such as 6x6) or repeats the previous pair of factors, such as 4 x 6 and 6 x 4.

If students are struggling, you may want to incorporate the use of area models, either on grid paper or created using square tiles, to show the relationships between the numbers. For example, with 12 +18, 12 can be represented as a 2 by 6 array and 18 as an 2 by 9 array, so that you can see that 12 + 18 is 6 twos and 9 twos (together, 15 twos). The same can be demonstrated using other common factors, as shown below.



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