

Transformations, Patterns, and Generalizations
Task 1: Say What you See Dog Task

Framework Cluster	Reasoning about Similarity and Transformations
Standard(s)	8.G.3 Describe the effect of dilations about the origin, translations, rotations about the origin in 90 degree increments, and reflections across the x-axis and y-axis on two dimensional figures using coordinates.
Materials/Links	Say What you See Dog Task Adapted from Which One Doesn't Belong?
Learning Goal(s)	Develop a shared understanding of vocabulary related to rigid and non-rigid transformations.

Task Overview:

This is an introductory task of the different types of transformations. Students may be familiar with the terms slide, turn, and flip but have not had detailed instruction regarding these terms as they relate to two-dimensional shapes on a coordinate grid. This task is similar to a “Number Talk” where students verbalize their current understandings of the concept and through discussion, build a deeper level of understanding. This task introduces vocabulary associated with rigid and non-rigid motions on a coordinate plane.

Prior to Lesson: This is an introductory task to the concept and vocabulary associated with transformation, so no prior knowledge is required. The follow-up lessons and tasks will refer back to prior knowledge gained in this task.

Teaching Notes:**Task launch:**

- Choose a task from the website, [Which One Doesn't Belong?](#), and use as an introduction to an exercise of this type. After asking students “Which One Doesn't Belong?” and having them explain their reasoning, teachers can pose further questions that require students to respond verbally to show their understanding. There should be a clear expectation that all ideas are accepted and can often lead to uncovering misconceptions. Possible questions: What do you notice? What is similar about these numbers (or pictures)? What is different about these numbers (or pictures)? What do some of the items have in common? Is there one that is different from all the others? If so, how?

Directions:

- Teacher will display the Say What You See Dog Task.
- Teacher will pose questions to the students in order to develop their reasoning to include appropriate vocabulary (transformation, translation, reflection, rotation, dilation).

Possible Strategies/Anticipated Responses:

Anticipated Responses	Teacher Follow-Up Questions
One of the dogs is brown.	Is there a relationship to the brown dog and any of the others?
One of the dogs is smaller.	<p>What is the mathematical word used to make an object larger or smaller? (Introduce “Dilation” and relate to scale factor from 7th grade.)</p> <p>What scale factor do you think was used to dilate the small dog to create the larger one?</p> <p>Are there two dogs that were dilated using a scale factor of 1?</p> <p>Do you see any dogs that were dilated using a scale factor < 1?</p>

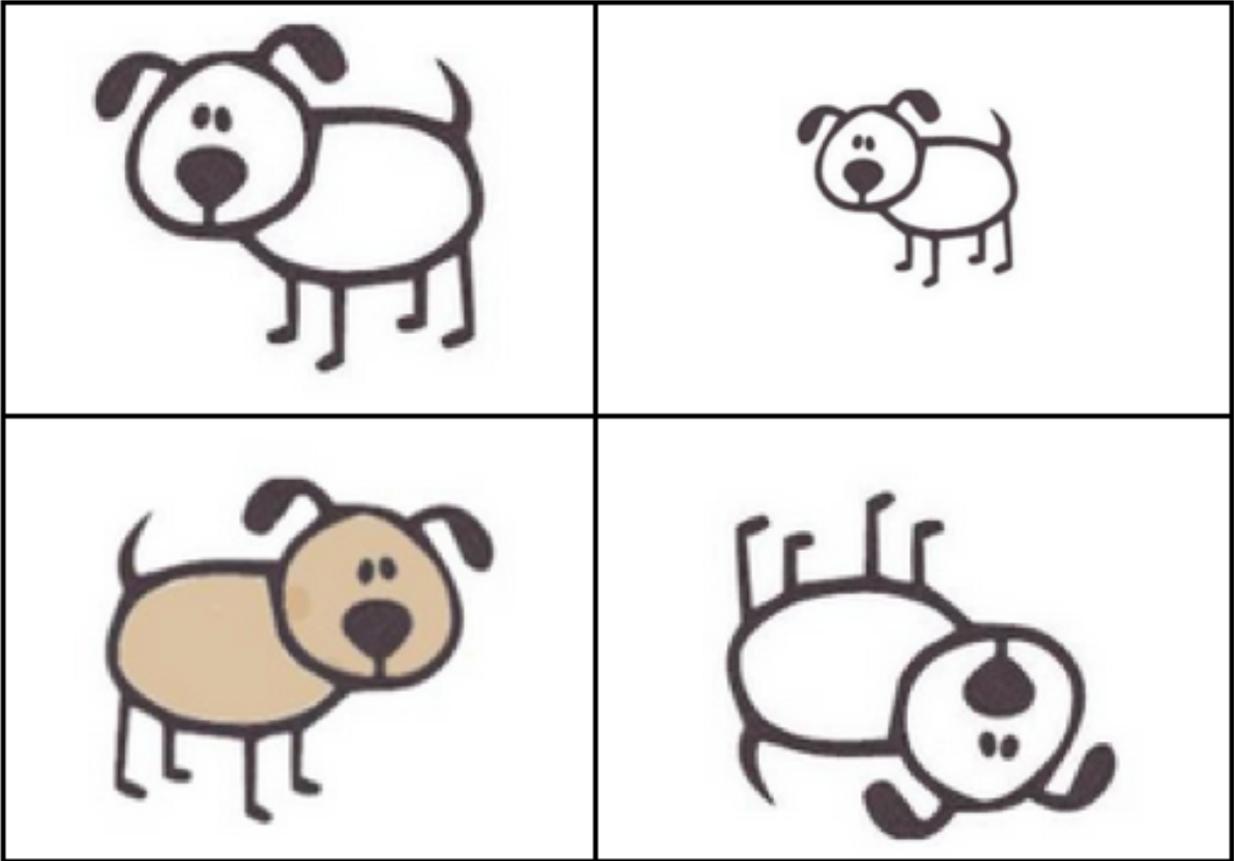
One dog has been flipped.	What is the mathematical term for flip or mirror image? (Introduce “Reflection” and probe further by asking if they see a reflection over a horizontal or vertical line.
One dog was turned.	What is the mathematical term for a turn? (Introduce the term “Rotation” and probe for student understanding of direction regarding rotations).
The brown dog was flipped then moved up.	What does “move up” mean? Introduce the term “Translation” and discuss how a translation occurs in the horizontal or vertical direction or both, resulting in a diagonal move.

Adapted from *Taking Action: Implementing Effective Mathematics Teaching Practices*, NCTM, 2017

Student sheets begin on next page.

Name _____ Date _____

Say What You See



Transformations, Patterns, and Generalizations Task 2: Translation Investigation	
Framework Cluster	Reasoning about Similarity and Transformations
Standard(s)	8.G.3 Describe the effect of dilations about the origin, translations, rotations about the origin in 90-degree increments, and reflections across the x-axis and y-axis on two dimensional figures using coordinates.
Materials/Links	Translation Investigation student handout paper/pencil
Learning Goal(s)	<ul style="list-style-type: none"> • Students translate two-dimensional figures on a coordinate plane, analyzing and reflecting on patterns they observe. • Students will describe the effect of a translation of a figure on a coordinate plane by connecting the observed patterns of counting on a grid to a general algebraic rule using its coordinates.
<p>Task Overview: The purpose of this investigation is for students to connect their understanding of a rigid transformation, specifically a translation, as a visual model to a more abstract, algebraic rule based on the patterns observed. The charts introduce using prime notation for the coordinates of the image(s) of a shape. Vocabulary terms important to use throughout this task are “pre-image” (original shape) and “image” (new shape).</p>	
<p>Prior to Lesson: From Task 1, students should have a basic understanding that a translation is a slide (vertically or horizontally) on or off a coordinate plane. More specifically, translation is a term used in geometry to describe a function that moves an object a certain distance. The object is not altered in any other way. It is not rotated, reflected or re-sized. Remind students that for a translation, every point of the pre-image must be moved in the same direction and for the same distance to create its image.</p>	
<p>Teaching Notes: Task launch:</p> <ul style="list-style-type: none"> • Ask students to think about the word “translate” in the following sentence. “I used my Spanish-English dictionary to <i>translate</i> the word ‘chocolate’ into Spanish.” Probing questions: - What does the word “translate” mean to you? - How can we relate it to shapes on a grid in mathematics? - Can you find any commonalities of how the word is being used? <p><i>(Origin: Middle English: from Old French, or from Latin <i>translatio(n)-</i>, from <i>translat-</i> ‘carried across’. The idea is that the word <i>translate</i> means to change in some form, usually to a different location or to a different text language, however, the object being translated retains the same characteristics or meaning.)</i></p> <p>Directions:</p> <ul style="list-style-type: none"> • Teachers should guide student thinking through the use of questions that facilitate observable patterns in their work. For instance, after shape 1 has been translated 2 units right and 3 up and the new coordinates are located on the graph and recorded in the chart, the teacher should ask students to verbalize any patterns they see with the original and new vertices for each x and y coordinate via a turn and talk strategy. Encourage students to write a rule based on the patterns they observe, knowing that the rule may be revised after more shapes are translated. • The teacher should then encourage students to reflect on the original and new vertices after shape 2 has been translated. <i>(Note that each new translation always refers back to the original pre-image).</i> This time, after a brief turn and talk, student pairs can share out to the whole class their observations. Again, students are encouraged to write a rule that applies to the coordinates for this translation. • Based on findings and reflection on understanding of work with shape 2, the teacher should ask students to determine and make any changes to the rule written for shape 1. 	

- By shape 3, most students should be able to apply the patterns of adding and subtracting to the x and y coordinates based on the transformation directions and confirm the vertices of the new image is correct with the graph.
- Finally, students should feel confident in their ability to create the vertices of a new image given the original vertices and translation directions. Teachers should confirm student algebraic results visually on smartboard or other classroom visual tool.

Possible Strategies/Anticipated Responses:

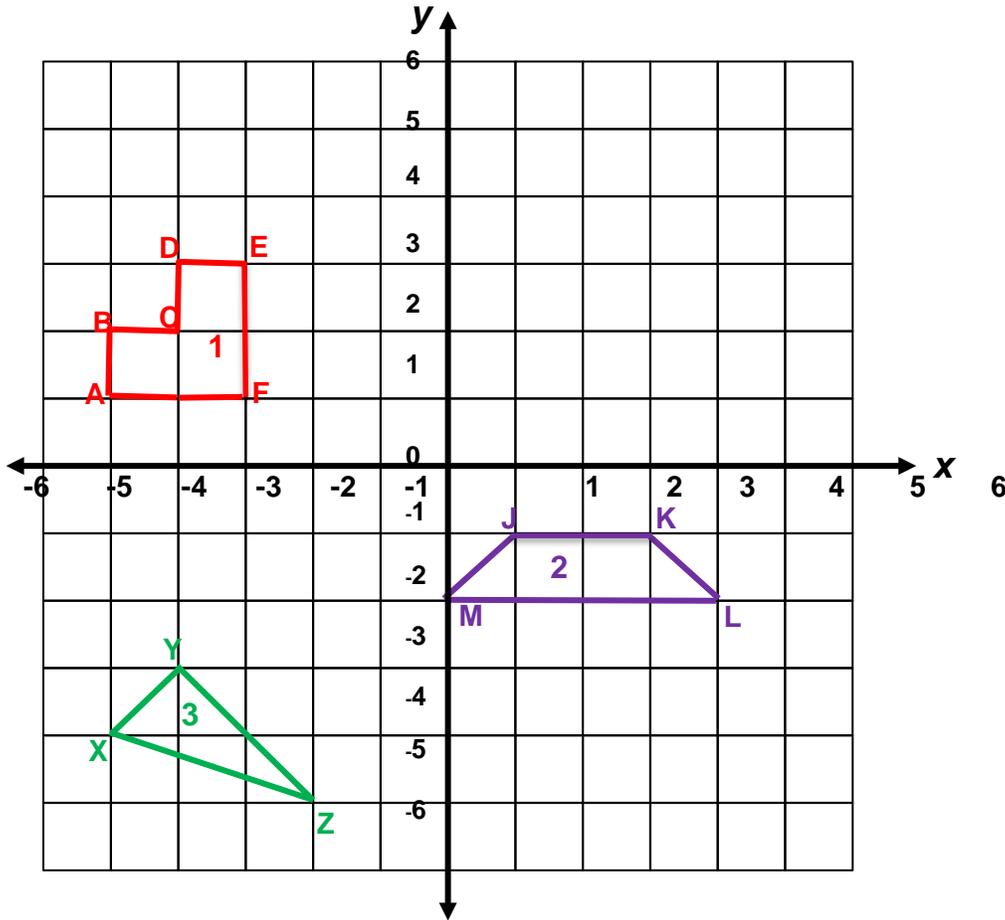
Possible Strategies: Based upon previous knowledge of the definition of a translation being a slide, students will initially count units on the coordinate grid to move the shapes according to directions.

Anticipated Responses:

Shape (pre-image)	Original (x, y)	On graph, translate 2 right and 3 up. Write new (x, y)	On graph, translate 1 left and 2 up. Write new (x, y)	Without graphing, write new (x, y) for a translation 4 right and 3 down
1	A (-5, 1)	A' (-3, 4)	A'' (-6, 3)	A''' (-1, -2)
	B (-5, 2)	B' (-3, 5)	B'' (-6, 4)	B''' (-1, -1)
	C (-4, 2)	C' (-2, 5)	C'' (-5, 4)	C''' (0, -1)
	D (-4, 3)	D' (-2, 6)	D'' (-5, 5)	D''' (0, 0)
	E (-3, 3)	E' (-1, 6)	E'' (-4, 5)	E''' (1, 0)
	F (-3, 1)	F' (-1, 4)	F'' (-4, 3)	F''' (1, -2)
	Rule	(x + 2, y + 3)	(x - 1, y + 2)	(x + 4, y - 3)
2	J (1, -1)	J' (3, 2)	J'' (0, 1)	J''' (5, -4)
	K (3, -1)	K' (5, 2)	K'' (2, 1)	K''' (7, -4)
	L (4, -2)	L' (6, 1)	L'' (3, 0)	L''' (8, -5)
	M (0, -2)	M' (2, 1)	M'' (-1, 0)	M''' (4, -5)
	Rule	(x + 2, y + 3)	(x - 1, y + 2)	(x + 4, y - 3)
3	X (-5, -4)	X' (-3, -1)	X'' (-6, -2)	X''' (-1, -7)
	Y (-4, -3)	Y' (-2, 0)	Y'' (-5, -1)	Y''' (0, -6)
	Z (-2, -5)	Z' (0, -2)	Z'' (-3, -3)	Z''' (2, -8)
	Rule	(x + 2, y + 3)	(x - 1, y + 2)	(x + 4, y - 3)

Student Sheet on the next page

Translation Investigation



Shape (pre-image)	Original (x, y)	On graph, translate 2 right and 3 up. Write new (x, y)	On graph, translate 1 left and 2 up. Write new (x, y)	Without graphing, write new (x, y) for a translation 4 right and 3 down
1	A (,) B (,) C (,) D (,) E (,) F (,)	A' (,) B' (,)	A'' (,) B'' (,)	A''' (,)
	Rule	(x + , y +)	(,)	(,)
2	J (,) K (,) L (,) M (,)			
	Rule	(,)	(,)	(,)
3	X (,) Y (,) Z (,)			
	Rule	(,)	(,)	(,)

Transformations, Patterns, and Generalizations
Task 3: Reflection Investigation

Framework Cluster	Reasoning about Similarity and Transformations
Standard(s)	8.G.3 Describe the effect of dilations about the origin, translations, rotations about the origin in 90-degree increments, and reflections across the x-axis and y-axis on two dimensional figures using coordinates.
Materials/Links	Reflection Investigation student handout paper/pencil
Learning Goal(s)	<ul style="list-style-type: none"> Students reflect two-dimensional figures on a coordinate plane, analyzing and reflecting on patterns they observe. Students will describe the effect of a reflection of a figure on a coordinate plane by connecting the observed patterns of counting on a grid to a general algebraic rule using its coordinates.

Task Overview:

The purpose of this investigation is for students to connect their understanding of a rigid transformation, specifically a reflection, as a visual model to a more abstract, algebraic rule based on the patterns observed. Vocabulary important to emphasize through the use of this task are “pre-image” (original shape) and “image” (new shape).

Prior to Lesson:

Students should have a basic understanding that a reflection is a flip across a line of reflection on or off a coordinate plane. More specifically, reflection is a term used in geometry to describe a function that creates a mirror image of the figure on the other side of the axis of reflection.

Teaching Notes:**Task launch:**

- The teacher can use a mirror in class and ask students to describe what the “reflection” of the teacher shows in the mirror. (Or provide a picture of someone looking in a mirror.)

Questions to ask: - Does the reflection seen in the mirror differ from the original? If so, how?

- Did the reflection change in size?
- What are characteristics of a reflection?

Directions:

- Two dimensional shapes in 8th grade are only reflected over the x or y-axis.
- Using the same turn and talk strategy described in the previous translation task, students should predict the general algebraic rule based on observable patterns when reflecting shape 1 and revise if needed after reflecting shape 2.
- Students should be able to verbalize their understanding that a reflection over the x-axis results in a coordinate where the x value remains the same but the y value is its opposite, or $(x, -y)$. Similarly, a reflection over the y-axis results in a coordinate where the x value becomes its opposite and the y value remains the same, or $(-x, y)$.
- Teachers should confirm student algebraic results visually on smartboard or other classroom visual tool for Triangle XYZ, where students are applying the algebraic rule without graphing.

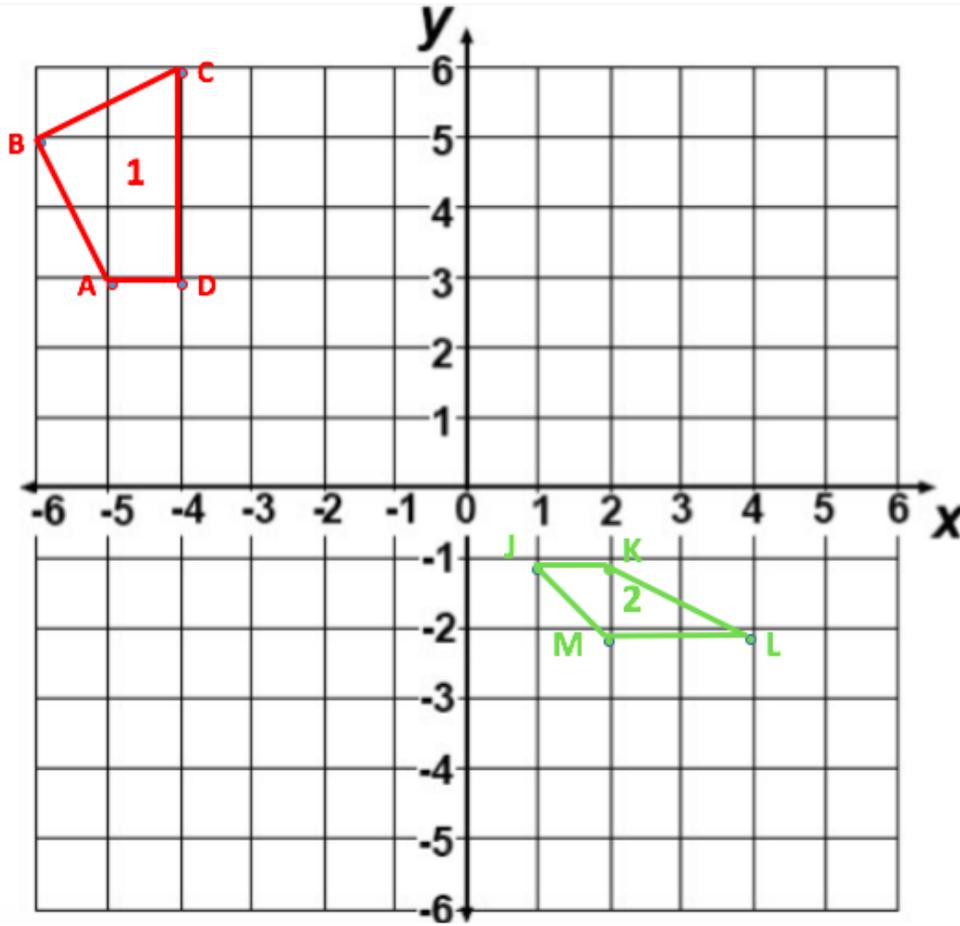
Possible Strategies/Anticipated Responses:

Possible Strategies: Applying previous understanding that a reflection is a mirror image or flip of a shape, most students will count the number of units each vertex is from the axis of reflection and count the same number of units in the opposite directions to locate the new vertex. However, the goal is again for students to move from being dependent on a visual coordinate grid to a more abstract understanding of how the coordinates change based on the reflection line.

Correct Table:

Shape (pre-image)	Original (x, y)	On graph, reflect over x-axis. Write new (x, y)	On graph, reflect over y-axis. Write new (x, y)
1	A (-5, 3) B (-6, 5) C (-4, 6) D (-4, 3)	A' (-5, -3) B' (-6, -5) C' (-4, -6) D' (-4, -3)	A'' (5, 3) B'' (6, 5) C'' (4, 6) D'' (4, 3)
	Rule	(keep x, opposite y) or (x, -y)	(opposite, keep y) or (-x, y)
2	J (1, -1) K (2, -1) L (4, -2) M (2, -2)	J' (1, 1) K' (2, 1) L' (4, 2) M' (2, 2)	J'' (-1, -1) K'' (-2, -1) L'' (-4, -2) M'' (-2, -2)
	Rule	(x, -y)	(-x, y)
		Without graphing, write new (x, y) for an x-axis reflection	Without graphing, write new (x, y) for a y-axis reflection
	If ΔXYZ has vertices: X (-2, 4) Y (-3, -5) Z (4, -1)	X' (-2, -4) Y' (-3, 5) Z' (4, 1)	X'' (2, 4) Y'' (3, -5) Z'' (-4, -1)
	Rule	(x, -y)	(-x, y)

Student sheets begin on next page.



Reflection Investigation

Shape (pre-image)	Original (x, y)	On graph, reflect over x-axis. Write new (x, y)	On graph, reflect over y-axis. Write new (x, y)
1	A (,) B (,) C (,) D (,)	A' (,) B' (,)	A'' (,)
	Rule		
2	J (,) K (,) L (,) M (,)		
	Rule		
		Without graphing, write new (x, y) for an x-axis reflection	Without graphing, write new (x, y) for a y-axis reflection
	If ΔXYZ has vertices: X (-2, 4) Y (-3, -5) Z (4, -1)	X' (,) Y' (,) Z' (,)	X'' (,) Y'' (,) Z'' (,)
	Rule		

Transformations, Patterns, and Generalizations Task 4: Rotation Investigation	
Framework Cluster	Reasoning about Similarity and Transformations
Standard(s)	8.G.3 Describe the effect of dilations about the origin, translations, and reflections across the x-axis and y-axis rotations about the origin in 90 degree increments, on two dimensional figures using coordinates.
Materials/Links	Rotation Investigation student handout paper/pencil
Learning Goal(s)	<ul style="list-style-type: none"> ● Students rotate two-dimensional figures on a coordinate plane, analyzing and reflecting on patterns they observe. ● Students will describe the effect of a rotation of a figure on a coordinate plane by connecting the observed patterns of counting on a grid to a general algebraic rule using its coordinates.
<p>Task Overview: The purpose of this investigation is for students to connect their understanding of a rigid transformation, specifically a rotation, as a visual model to a more abstract, algebraic rule based on the patterns observed. Vocabulary important to emphasize through the use of this task are “pre-image” (original shape) and “image” (new shape).</p>	
<p>Prior to Lesson: Students should have a basic understanding that a rotation is a turn on or off a coordinate plane.</p>	
<p>Teaching Notes: Task launch:</p> <ul style="list-style-type: none"> ● Using an analog classroom clock, ask students if they can give a time based on the following criteria assuming the center of the clock is the center of rotation: <ul style="list-style-type: none"> ○ Can you give a time on the clock that displays a 90 degree angle? Are there others? ○ Can you give a time on the clock that displays a 180 degree angle? Are there others? ○ Can you give a time on the clock that displays a 270 degree angle? Are there others? ○ If 3:00 is assumed to be a 90 degree rotation clockwise, what would 9:00 represent? ○ Are there multiple ways to represent the same time? <p>Directions:</p> <ul style="list-style-type: none"> ● Rotations of two-dimensional figures in 8th grade are always centered at the origin and only occur in increments of 90 degrees. ● If students have never been exposed to rotating a point or shape on a grid, some prior work may be needed to help students visualize the degrees of rotation (only increments of 90), and direction (clockwise and counter-clockwise). <i>Hint: The corner of a sheet of paper (or index card) is a great tool to use when helping students visualize the new location of a point. Place the corner of the paper at the origin and use one side of the paper to connect (pass through) the original point. Then, the other side of the paper from the corner is the invisible line that must contain the new point after a 90 rotation. This can be used for any direction and actually builds understanding that 90 cw would be the same as 270 ccw. The same idea can be used to visually model a 180 rotation by using one edge of the paper to connect the original point to the origin and then following the edge of the paper in the opposite direction to create the invisible line that the new point should be located.</i> ● Using the same turn and talk strategy described in the previous tasks, students should predict the rules based on observable patterns after rotating shape 1 and revise if needed after shape 2. ● Students should be able to verbalize their understanding for each rotation and write the new 	

coordinates appropriately.

- Teachers should confirm student algebraic results visually on smartboard or other classroom visual tool for Triangle XYZ, where students are applying the algebraic rule without graphing.
- The general algebraic rules for rotations can be confusing for students to try to memorize, and memorization is not the goal. However, students should be able to create the general rules for each rotation based on their visual understanding of how the shapes are turning and using the signs of coordinates based on quadrant locations.

Degree of Rotation	Clockwise	Counter-Clockwise
90	$(y, -x)$	$(-y, x)$
180	$(-x, -y)$	$(-x, -y)$
270	$(-y, x)$	$(y, -x)$
360	(x, y)	(x, y)

Possible Strategies/Anticipated Responses:

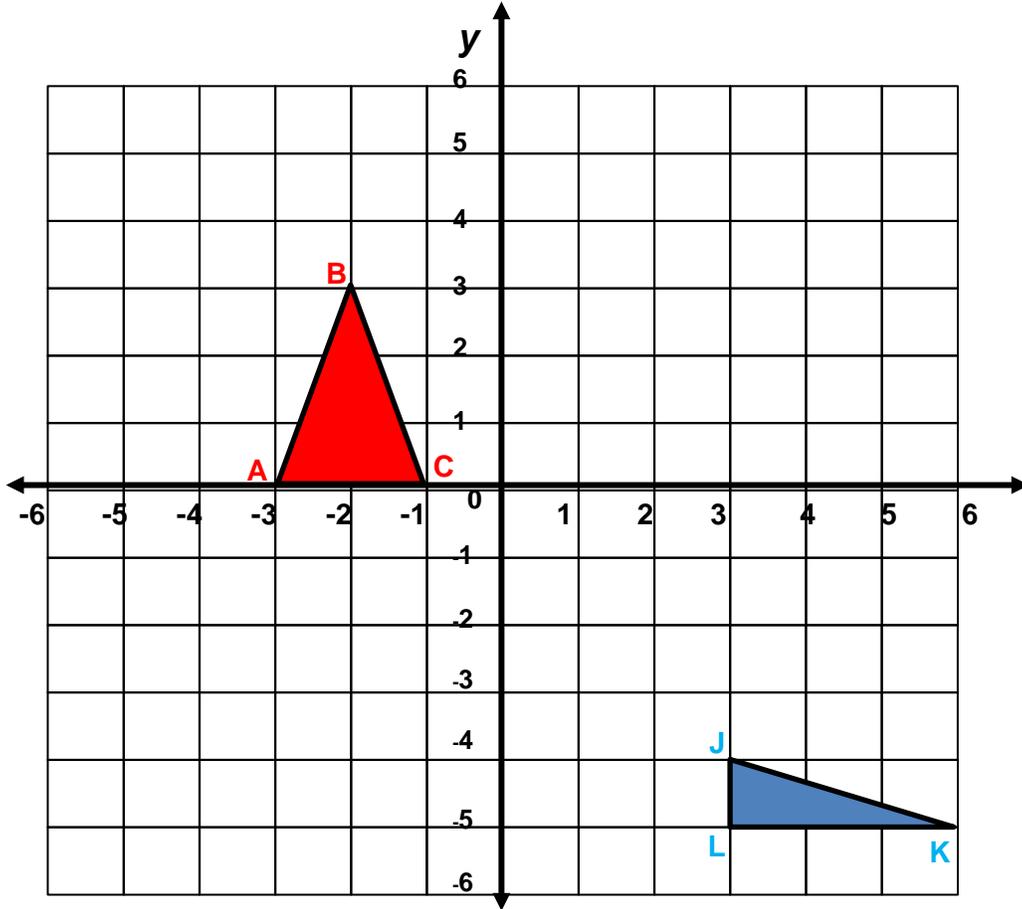
Possible Strategies: If students simply try to memorize the general rules for rotations, they often get confused on whether the order of the x and y change, the sign changes, or both. Have students visualize what quadrant the image of a shape will be located after a specific rotation, and use the signs of points in that quadrant to help them.

Shape (pre-image)	Original (x, y)	On graph, rotate 90° counterclockwise Write new (x, y)	On graph, rotate 180° Write new (x, y)	On graph, rotate 270° counterclockwise Write new (x, y)
1	A (-3, 0) B (-2, 3) C (-1, 0)	A' (0, -3) B' (-3, -2) C' (0, -1)	A'' (3, 0) B'' (2, -3) C'' (1, 0)	A''' (0, 3) B''' (3, 2) C''' (0, 1)
	Rule	(opposite y, x value) or (-y, x)	(opposite x, opposite y) or (-x, -y)	(y value, opposite x) or (y, -x)
2	J (3, -4) K (6, -5) L (3, -5)	J' (4, 3) K' (5, 6) L' (5, 3)	J'' (-3, 4) K'' (-6, 5) L'' (-3, 5)	J''' (-4, -3) K''' (-5, -6) L''' (-5, -3)
	Rule	(-y, x)	(-x, -y)	(y, -x)
		Without graphing, write new (x, y) a 90° CCW rotation	Without graphing, write new (x, y) for a 180° rotation	Without graphing, write new (x, y) for a 270° CCW rotation
	If ΔXYZ has vertices: X (-2, 4) Y (-4, 5) Z (1, 6)	X' (-4, -2) Y' (-5, -4) Z' (-6, 1)	X'' (2, -4) Y'' (4, -5) Z'' (-1, -6)	X''' (4, 2) Y''' (5, 4) Z''' (6, -1)
	Rule	(-y, x)	(-x, -y)	(y, -x)

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Name _____ Date _____

Rotation Investigation



Shape (pre-image)	Original (x, y)	On graph, rotate 90° counterclockwise Write new (x, y)	On graph, rotate 180° Write new (x, y)	On graph, rotate 270° counterclockwise Write new (x, y)
1	A (,) B (,) C (,)	A' (,) B' (,)	A'' (,)	A''' (,)
	Rule			
2	J (,) K (,) L (,)			
	Rule			
		Without graphing, write new (x, y) a 90° CCW rotation	Without graphing, write new (x, y) for a 180° rotation	Without graphing, write new (x, y) for a 270° CCW rotation
	If ΔXYZ has vertices: X (-2, 4) Y (-4, 5) Z (1, 6)			
	Rule			