Creating A Triangular Flag

Frameworks Cluster	Reasoning with Area and Surface Area
Standards	**NC.6.G.1** *Create geometric models to solve real-world and mathematical problems to:* ● *Find the area of triangles by composing into rectangles and decomposing into right triangles.* ● *Find the area of special quadrilaterals and polygons by decomposing into triangles or rectangles.* **SMP 1** *Make sense of problems and persevere in solving them* **SMP 5** *Use appropriate tools strategically* **SMP 6** *Attend to precision* **SMP 7** *Look for and make sense of structure*
Materials/Links	● Any 9 x 12 construction paper (or other paper cut to size), rulers, scissors, flag, notebook paper for letter ● Olympic Flag image: https://www.britannica.com/topic/flag-of-the-Olympic-GamesOne ● One copy of "Creating a Triangular Flag" handout per student
Learning Goal	Students will determine a base and height combination that produces a triangle within a given range, and justify their solution by decomposing and/or composing with right triangles.

Task Overview:

- In this hands-on task, students will determine appropriate dimensions and create a triangle-shaped flag that falls in a given range of areas, then justify the accuracy of their solution.

Prior to Lesson:

- Students should have previously completed the *Areas of Triangle* task (prior task in this sequence), in which they decomposed and compared the areas of various triangles.
- Students should be familiar with finding area of right triangles and finding the area of equilateral and isosceles triangles by decomposing into right triangles or composing into rectangles.
- Note that at this point in the year, students have not learned multiplication of a mixed number by a mixed number or a decimal by a decimal.
- Be sure to remind students that length is measured in units, while area is measured in square units. Give them exposure to various ways of expressing area (square inches, sq. in., in^2)

Teaching Notes:

Vocabulary for this lesson*: length, width, area, base, height, dimension, isosceles triangle, right triangle*

Task launch:

- Hook students by showing them this 2018 Winter Olympics Opening ceremony clip, (https://www.youtube.com/watch?v=e7IfDcE1iRg) being sure to point out the Olympic flag and the first countries' flags being carried in. Which flag is your favorite? What is the purpose of a flag? (students might suggest pride as a reason which can carry over into the reason you want to decorate a new flag).

Directions:

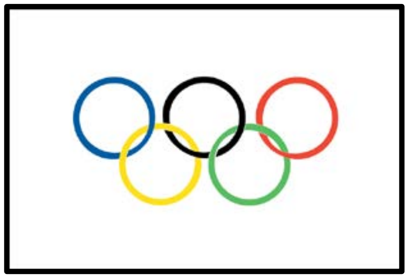
- Distribute the handout and give the class time to understand what they are being asked to do (*not* how to accomplish the task). They need to understand that the flag they design must be within the given area range, and the dimensions must fit within the 9x12 paper size. Be careful not to lead students' thinking or to let them give away too much of their own thinking when discussing as a group.
- Ask students to plan their flag on the back of the worksheet or scrap paper before they begin working on the construction paper. Consider waiting to give them construction paper until after you approve their draft.

- Possible questions for students as they work:
  - Do you know the area of the construction paper?
  - How is finding the area of a rectangle and triangle similar? How is it different?
  - What is the largest (or smallest) base (or height) possible for the flag? (limit to whole numbers)
  - For students who are still having trouble with finding the area of *right* triangles, you might guide them through the process of finding the area of a *right* triangle outside of the given range (for example, 10 in x 5 in = 50/2 = 25 in^2)
- Extension Questions (optional):
  - List all of the possible base/height dimensions of a flag that is 40 square inches. How do you know you've listed them all?
  - What is the greatest possible area for a flag between 40 and 60 square inches, if the base and height must be whole numbers? How do you know it's the largest possible?
  - Is it possible to create a flag that would have the same area as the area of the wasted fabric? Explain and justify your answer.
  - If the dimensions of the triangle must be whole numbers, list the base and height of as many triangles as you can that would fall within the range of 40 to 60 in^2.
  - What are the dimensions of a flag that is closest to 50 in^2 without being exactly 50 in^2? (use any number desired)
- After students have created their flags, summarize the lesson through a whole-group discussion of various areas found, and different dimensions for the same areas found. Consider using the Class Discussion Planner (https://tinyurl.com/discussion-planner) as a guide.
- *Task follow up:* After students have had sufficient practice with area of a triangles and parallelograms, task can be followed up by *Formative Assessment: Grandfather Tang's Puzzle*

Lesson plan template adapted from *Taking Action: Implementing Effective Mathematics Teaching Practices*, NCTM, 2017

Name ____ Date _______________________

Creating a Triangular Flag



Since the 1914 Olympic Games, the Olympic flag has been flown in opening ceremonies. The flag is a rectangular white flag with five interlocking rings, representing the five "continents" that participated in the games: Africa, the Americas, Asia, Europe, and Oceania. After over a century of this flag, the Olympic design committee feels that a special flag should be created to be flown in closing ceremonies of the 2022 Olympics.

The school Olympic master has asked you to determine possible dimensions for flag that is an <u>isosceles</u> triangle with an area between 40 and 60 in^2.

<u>Part 1:</u>

1. Before you cut or write on the construction paper, use the back of this paper to plan your flag design.
2. Then, use the *9 inch x 12 inch* construction paper to create your flag. Use a ruler to draw the edges of your flag.
3. After you have designed your flag, record the base, height, and area below, and carefully cut out your flag.

   a. The base of your flag = ____________ inches

   b. The height of your flag = __________ inches

   c. The area of your flag = _____________ square inches

<u>Part 2:</u> (Answer on the back of this paper or another sheet of paper.)

1. How do you know your flag is between 40 and 60 square inches? *Use what you know about right triangles to justify your answer.*

2. What is another possible base and height combination you could use to create a flag that meets the criteria?

3. If the actual flags that will be produced are cut from a 9 x 12 inch pieces of cloth, and the extra cloth is thrown away, how much cloth will be wasted if your design is used?

4. On a sheet of lined paper, write a brief letter to the Olympic Headquarters Association explaining how you decomposed the original rectangle (paper) to create your triangular shaped flag.

Possible Strategies/Anticipated Responses:

- Students often confuse the area of rectangle formula with the area of a triangle formula, forgetting to divide by 2.
    - Example: base = 5 in., height = 9 in. area = 45 in^2
- Students may not realize that the height must be perpendicular to the base, instead using the base and side length to find the area of the triangle.
- Students may create a right or scalene triangle rather than an isosceles triangle.
- Other combinations are possible, but one dimension (base or height) cannot exceed 9 inches and the other dimension cannot exceed 12 inches.
- The area should be between 40 in^2 and 60 in^2. *Some* possibilities are listed below. (Note: you may choose to limit your students to only whole numbers for the base and height, but some students may be ready to use fractions or decimals in their calculations)

Base	Height	Area of flag	Wasted fabric (paper)
8 in	10 in	(8 x 10)/2 = 40 in^2	108 - 40 = 68 in^2
8 in	10.5 in	(8 x 10½)/2 = 42 in^2	108 - 42 = 66 in^2
8 in	11 in	(8 x 11)/2 = 44 in^2	108 - 44 = 64 in^2
8 in	11.5 in	(8 x 11½)/2 =46 in^2	108 - 46 = 62 in^2
8 in	12 in	(8 x 12)/2 = 48 in^2	108 - 48 = 60 in^2
9 in	10 in	(9 x 10)/2 = 45 in^2	108 - 45 = 63 in^2
9 in	10.5 in	(9 x 10½)/2 = 47¼ in^2	108 - 47¼ = 60¾ in^2
9 in	11 in	(9 x 11)/2 = 49½ in^2	108 - 49½ = 58½ in^2
9 in	11.5 in	(9 x 11½)/2 = 51¾ in^2	108 - 51¾ = 57¼ in^2

- For the largest possible flag, students may recognize that the sides of the paper constraining the dimensions (9 x 12 inches). This produces the largest possible area, 54 in^2.

- For the smallest possible flag, 40 in^2, if base and height are limited to whole numbers, the only dimensions possible are 8 inches and 10 inches. Other options, such as 5 inches and 16 inches, are ruled out due to the size of the original paper.

Area of Triangular Gardens

Frameworks Cluster	Reasoning with Area and Surface Area
Standards	**NC.6.G.1** *Create geometric models to solve real-world and mathematical problems to:* • ~~*Find the area of triangles by composing into rectangles and decomposing into right triangles.*~~ • *Find the area of special quadrilaterals and polygons by decomposing into triangles or rectangles.* **SMP 1** *Make sense of problems and persevere in solving them* **SMP 5** *Use appropriate tools strategically* **SMP 6** *Attend to precision* **SMP 7** *Look for and make sense of structure*
Materials/Links	Student pages
Learning Goal	Using properties of area of triangles from previous tasks, students will find the area of parallelograms informally and then analyze students' work to form the formula.

Task Overview: Students use what they know about area of triangles to find the area of parallelograms on grid paper. Once they have found those areas, students will have a whole class debrief in which they share a variety of strategies. Then, they will move to the second page in which they will analyze two students' strategies for formalizing the area of a parallelogram. In this way, they will be encouraged to create the formal equation.

Prior to Lesson: Students should have worked with finding the area of triangles and rectangles. They should be familiar with using rectangles and to find the area of a triangle and vice versa.

Teaching Notes:

Launch:

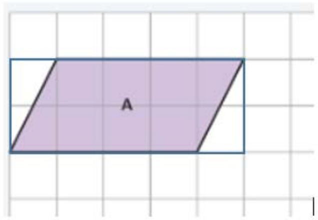
Remind students about the work they did with triangular and rectangular area relationships (i.e., that two triangles compose to create a rectangle and vice versa). Encourage students to use that knowledge to find the area of the parallelograms below. Let students know that they can cut the parallelograms any way that will help them.

Monitor Student Thinking
Students can try this individually and then talk to partners or work with partners from the beginning. As you walk around, spend only 2-3 minutes per group and collect data on who is using the strategies anticipated below.

Debrief Discussion
After students have completed the first page, ask students to present the different methods. Highlight those that "cut" off the triangle and move it as they will see that on the next page as one of the strategies. After the discussion have them work the second page with a partner and share different formulas.

Anticipated Student Reasoning

Strategy One: *Rectangle* Some students may draw a rectangle around the shape, find its area and subtract the two triangles' areas on the end.

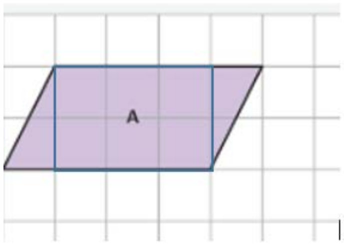


5 x 2 = 10 rectangle

Put both white triangles together to get 2 squares.

10 – 2 = 8.

Strategy Two: *Outer Triangles* Some students may find the area of the rectangle inside and then find the area of each triangle on the outside.

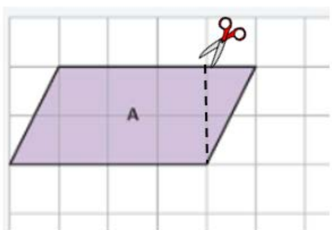


3 x 2 = 6 rectangle

Put both purple triangles together to get 2 squares.

6 + 2 = 8.

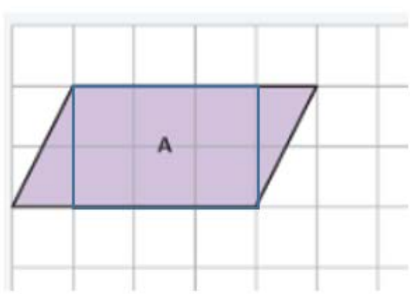
Strategy Three: *Triangles Cut and Paste* Some students may mentally cut the triangle off one end, move it to the other triangle and find the area of the newly formed rectangle.



4 x 2 = 8 rectangle

Cut purple triangle and move it to the left side to make a 4x2 rectangle

Strategy Four: *Estimation* Some students may find the area of the rectangle and then estimate the number of squares in the triangles on the ends.

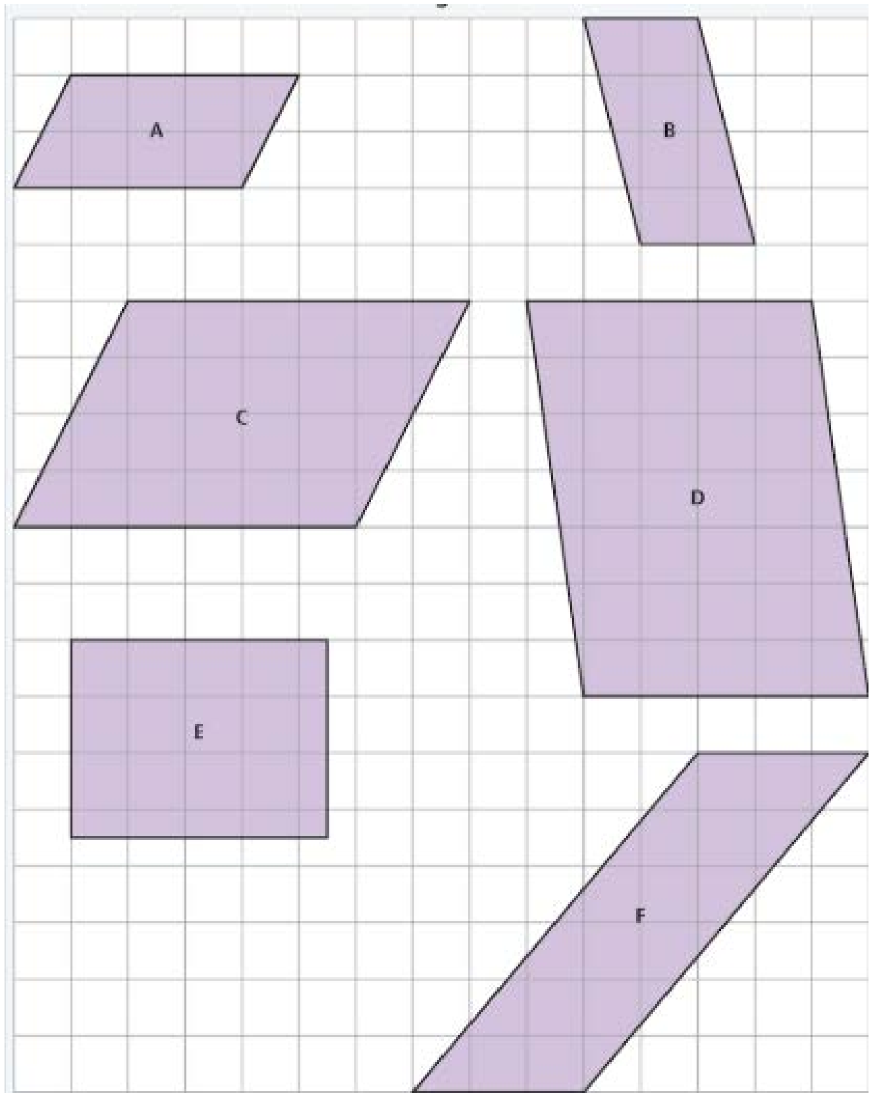


3 x 2 = 6 rectangle

Estimate the two triangles on the end to be 1 square each.

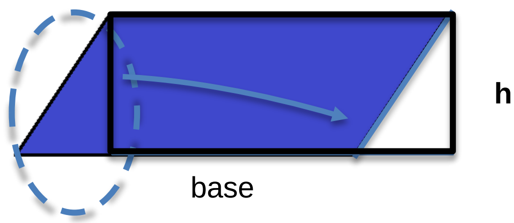
6 + 2 = 8

1. **For each of the parallelograms below, find the area (squares). You may draw on the inside or outside of each shape if it helps.**

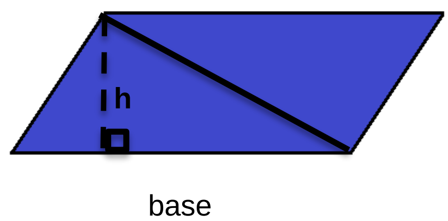


2. **Ruth and Tomás each invented a strategy for find the area of a parallelogram.**

RUTH'S Rectangle Strategy   A = b x h



TOMÁS' Triangle Strategy   A = ½ b h (2)



Who is right? Explain.

Mosaic Medallion

Frameworks Cluster	Reasoning with Area and Surface Area
Standard(s)	**NC.6.G.1** *Create geometric models to solve real world and mathematical problems to:* • ~~*Find the area of triangles by composing into rectangles and decomposing into right triangles.*~~ • *Find the area of special quadrilaterals and polygons by decomposing into triangles or rectangles*. **SMP 1** *Persevere through problem solving* **SMP 6** *Reason abstractly and quantitatively* **SMP 7** *Make sense of structure*
Materials/Links	• One copy of the task sheet and the large medallion sheet for each pair of students (or each student, if you plan to have students complete the task individually) • Paper square for folding (post it note, patty paper, origami paper, or 8 ½ inch squares cut from copy paper), one for each student • Extra copies of the medallion (you will need a supply of these so that students feel free to cut from them and manipulate the shapes they cut) • Photos of floor medallions (this could be an opportunity for a cross discipline tie-in with social studies, with ancient art and architecture) • Scissors, markers • Calculators • *Optional: hexagonal ceramic floor tiles can be purchased from a home improvement store for about 25¢ per tile. Students can use dry erase markers to draw on these as they strategize.*
Learning Goals	• Students will find the area of a hexagonal design by decomposing the hexagon into familiar shapes and finding their areas.

Task Overview:

- In this problem-based task, students will use their knowledge about the structure of a regular hexagon and recognize other shapes (triangles, rectangles, trapezoids) that could be created by decomposing the hexagon. Students will determine a way to decompose the hexagon into smaller shapes and will use the area of those shapes to find the area of the composed hexagon.

Prior to Lesson:

- Students should have had experience with math exercises that involve finding the area of rectangles, parallelograms and triangles
- Students should have had experience looking for shapes within shapes in order to find the area of the original shape. These experience should include opportunities to draw the outlines of the shapes they find, to fold the shape into its smaller shapes, and to cut the shape into its smaller shape parts in order to compose other familiar shapes.
- Plan student partner assignments in advance, if you plan to use this as a collaborative task.
- Students should be familiar with these terms: *regular polygons, diagonals, equilateral triangles, height, base, width, length, inscribed*

Teaching Notes:

- Teachers are strongly encouraged to work through the lesson activity prior to looking at the possible strategies in order to more deeply understand the student experience and anticipate responses and misconceptions.

Task launch:

- Show students pictures of tiled floors that have a medallion in the center. (Some may be found at this site: http://www.medalliondepot.com/gallery/)

- Remind students of the term “regular” polygon and its meaning: *a polygon in which all sides and all angles are congruent.*
- Show them a rectangle and ask the question: *What would a “regular rectangle” look like?* (a square)
- Give each student a square paper (post it note, patty paper, or 8½ inch square cut from copy paper)
  - Have students fold the square along <u>one</u> diagonal, then unfold to return to the square shape. Ask the question: *What statements about area could you make when a square is folded on its diagonal?* (the area of each triangle is ½ of the area of the square, the area of the square is two times the area of each triangle)
  - Have the students fold the square along the other diagonal, then unfold to return to the square shape. Ask the question: *What statements about the area could you make now?* (the area of each triangle is ¼ of the area of the square, the area of the square is four times the area of each small triangle, the area of two of the small triangles is equal to the area of one of the larger triangles, etc.)
  - If time allows, the YouCubed task “Paper Folding” could be used to further investigate area with folded squares and develop students’ ability justify their thinking: https://www.youcubed.org/tasks/paper-folding/
- Discuss and then display for reference: “A **hexagon** is a flat shape, all in one plane, with six sides all of equal length. The **hexagon** can be cut into six congruent, equilateral triangles.”

Monitor the task:

- Ask questions to scaffold for students who need support to proceed through the task. Use these questions sparingly and allow for adequate think time on the part of the student to avoid giving the task away.
  - What shapes do you know how to find the area of? (triangles and rectangles)
  - How might you fold the hexagon to create different shapes? (look for triangles, rectangles, trapezoids)
  - What shapes were created when you folded the hexagon? (answers will vary based on the orientation of the fold)
  - Is there another way to fold the shape to help you solve the problem?
  - What shapes are created when you draw one diagonal which is a line of symmetry? (two congruent trapezoids) Two diagonals?(two congruent equilateral triangles and two congruent rhombuses)
  - If you draw all the diagonals that divide the hexagon into two halves, what do you notice? (They meet at the center of the hexagon and decompose the hexagon into 6 congruent equilateral triangles)
  - Do you know the length of that diagonal? (180 cm) What else do you know? (half of the diagonal is 90 cm and that is the measure of the sides of the triangles)
- An extension activity is provided for students ready for more.

Summarize:

- After students have completed the task, summarize the lesson through a whole-group discussion of various strategies students used to decompose the hexagon, making connections between the strategies when possible. Consider using the Class Discussion Planner (https://tinyurl.com/discussion-planner) as a guide.
- An Exit Ticket (optional) is provided to assess students’ understanding. You may opt to only use one or two questions.

The “medallion” image is a mandala coloring pattern taken as a free download from this site: http://amind.co.kr/bbs/board.php?bo_table=hexagonmandala&wr_id=8&ckattempt=1

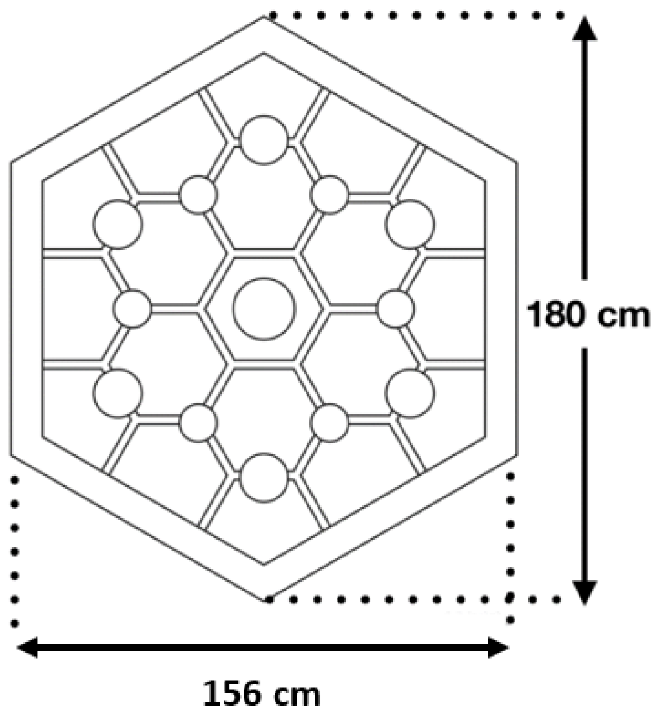
Lesson plan template adapted from *Taking Action: Implementing Effective Mathematics Teaching Practices*, NCTM, 2017

Student sheets begin on next page.

Name: ______________________________________ Date: ____________________

What is the area of the Mosaic Medallion?

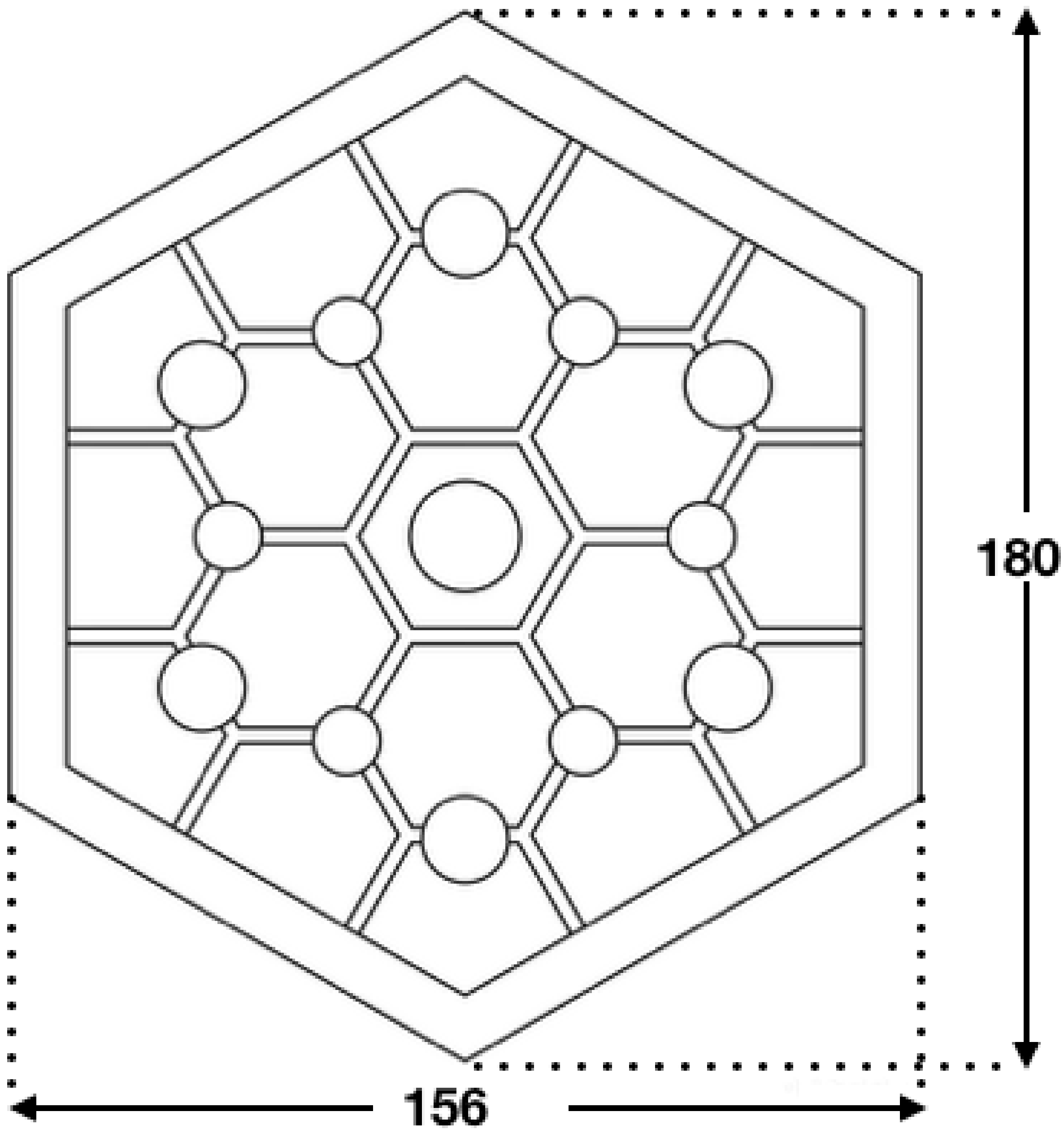
Task: You have been commissioned to work for a tile company. The old Mount Mourne National Bank has been demolished to be replaced with a newer more modern building. Before the demolition, a large mosaic tile medallion was removed so that it could be saved and installed in the new building. Your company would like to bid on the job of installing the medallion in the new building. In order to give the contractor an estimate for the cost of installation, your manager needs to know the area of the mosaic. Below is a diagram of the mosaic with the measurements sent from the contractor.

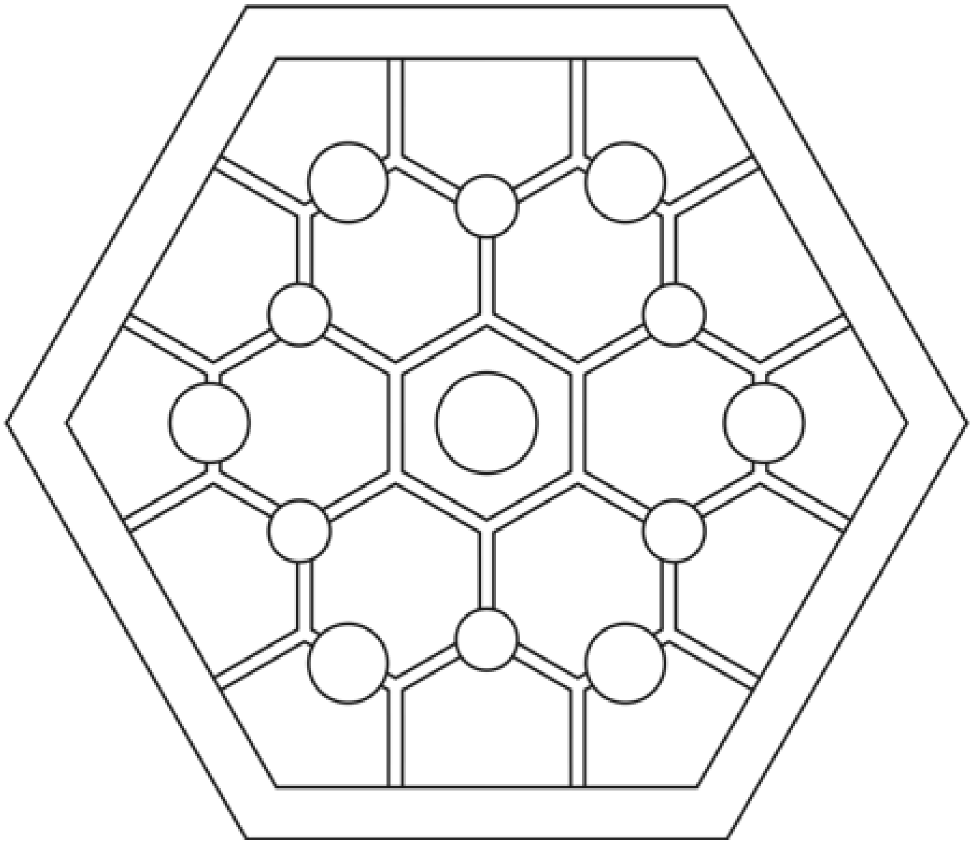


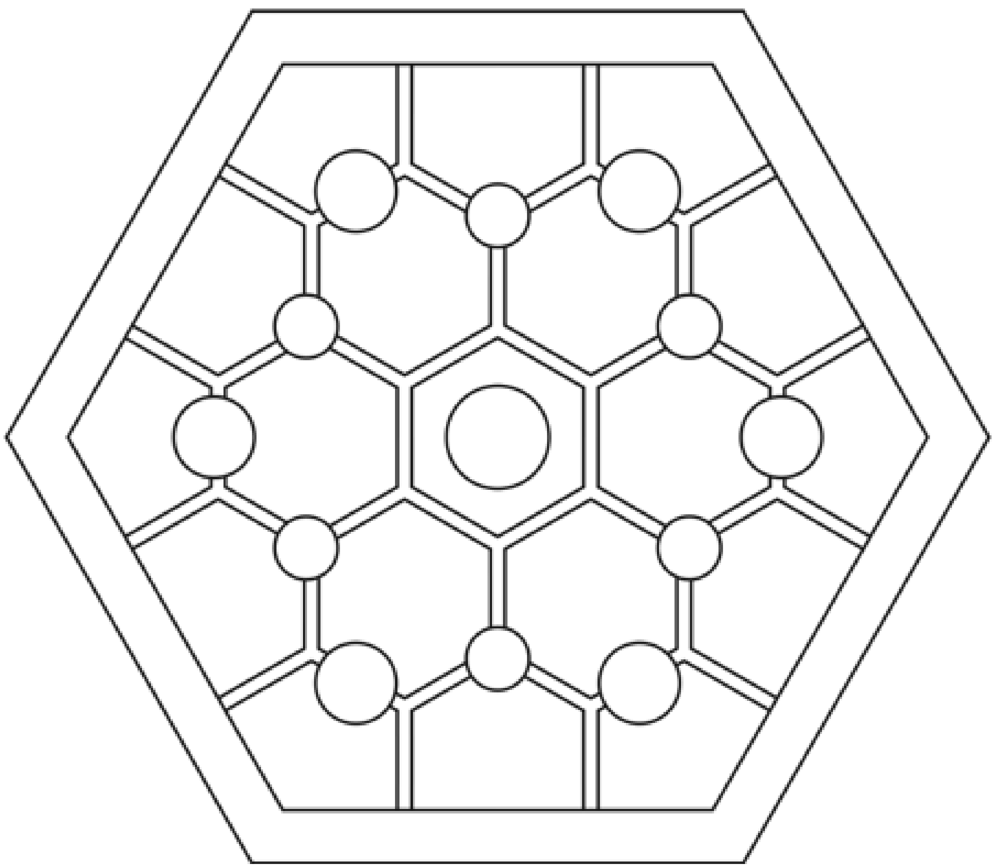
Using what you know about regular hexagons, and computing the area of rectangles and triangles, *find the area of the mosaic medallion*.

(You may use the large copies of the medallion which are on the next two pages to help you with your work.)

You must explain the strategy you used to determine the area of the regular hexagon.


180
156



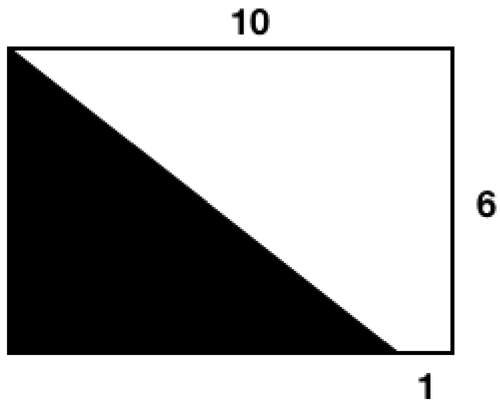


Name: ______________________________________ Date: ____________________

Exit Ticket:

Use the strategies you have used in the Mosaic task to find the area of the shaded regions of the following rectangles. Show how you arrived at your answer and express your answer in square units.

1.



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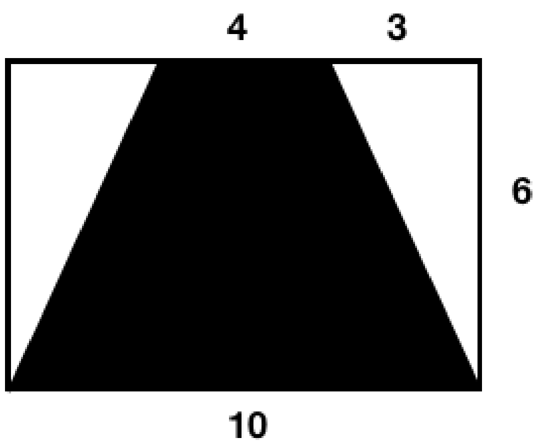
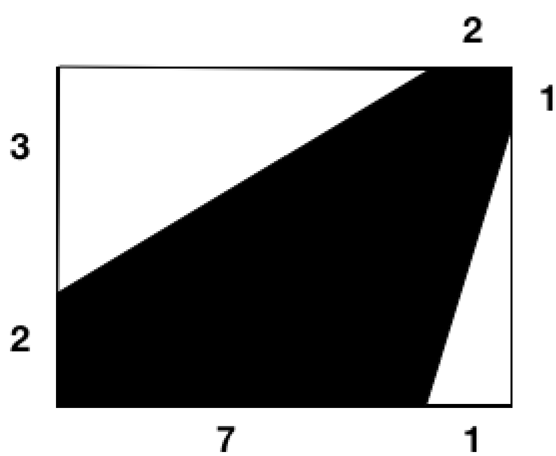


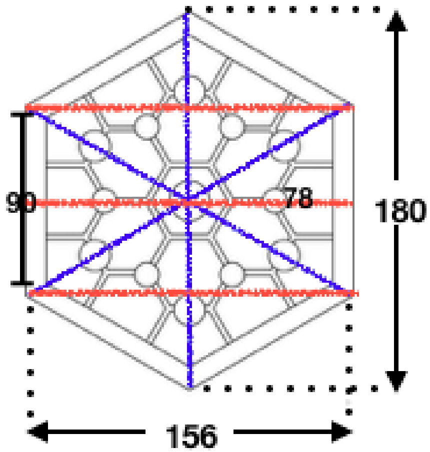
3.



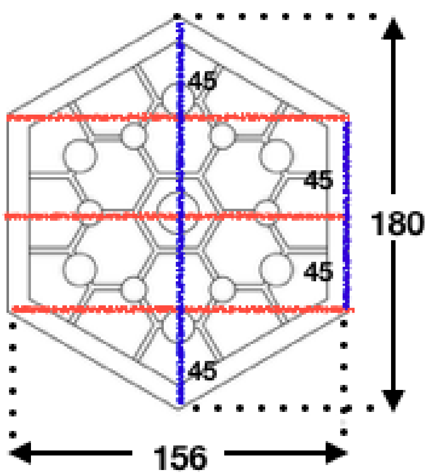
Possible Strategies/Anticipated Responses:

Note that the horizontal dimension given is approximate, but very close to the actual dimension that could be determined using the Pythagorean Theorem.

Anticipated Correct Solutions



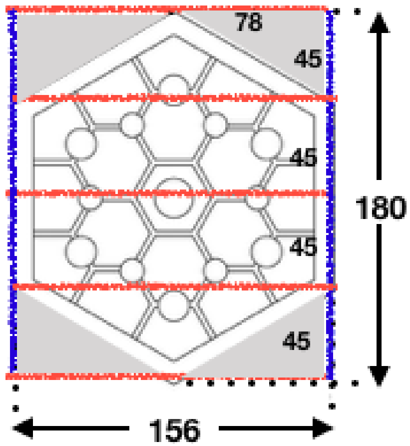
This student folded the hexagon along the three diagonals shown (blue) and saw that the length of each of those would match the vertical dimension given, 180. These folds divided the hexagon into six congruent, equilateral triangles. Since two of the sides of each triangle are each half of the diagonal folds, their lengths are both 90. (180/2 = 90)
The orange lines drawn correspond to the 156 cm horizontal dimension given. Half the measure of those lines (156/2) is equal to 78.
Therefore, each equilateral triangle has base = 90 cm and height = 78 cm.
The area of each triangle is ½(b)(h), and ½(90)(78) = 3,510.
Since the hexagon is composed of 6 of those triangles, its area is (6)(3510) = 21,060 cm^2.



This student saw the hexagon as a rectangle with triangles above and below.
The length of the rectangle is the horizontal dimension given, 156.
The width of the rectangle can be found when the drawn diagonal is divided into four equal lengths of 45, and the rectangle's width is 2 of these sections (45)(2)= 90.
The area of the rectangle is (156)(90) = 14,040 cm^2
The area of the isosceles triangles can be found in one of two ways:

1. Decompose each isosceles triangle into two right triangles. The base of each of these triangles is half of the horizontal dimension given, 156/2= 78, and the height is 45, so the area of each right triangle is (78)(45)/2 = 1755. Since there are four of these triangles, the area of all triangular sections is (4)(1755) = 7020.
2. Find the area of each isosceles triangle by multiplying the base length, 156, by the height of 45 and dividing by 2, (156)(45)/2 = 3,510. Since there are two of these triangles, the area of the triangular sections is (2)(3510) = 7020 (Some students might note that the division by 2 is not necessary, since there are two of the isosceles triangles.)

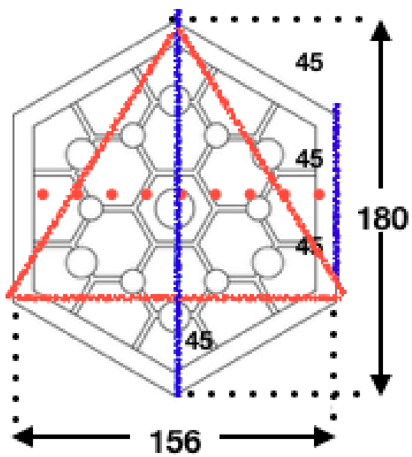
The area of the rectangle plus the area of the triangles gives the area of the hexagon: 14,040 + 7020 = 21,060 cm^2.
(An observant student might see that the two right triangles on the bottom can be rearranged to form the corners of a 45 x 156 rectangle on the top of the figure, which would have an area of 7,020. The two rectangles together have an area of 14,040 + 7,020 = 21,060 cm^2.)



This student saw the hexagon inscribed within a 156 X 180 rectangle, with four right triangles removed. The area of the large rectangle was computed: (156)(180) = 28,080 cm^2.
By folding in half and then in half again, the 180 cm side can be divided into four equal segments of 45 cm, which is the length of the base (or height) of the missing triangles. The height (or base) of this right triangle would be 78, half of the 156 cm horizontal dimension given.
Therefore, the area of each triangle would be (45)(78)/2 = 1755.

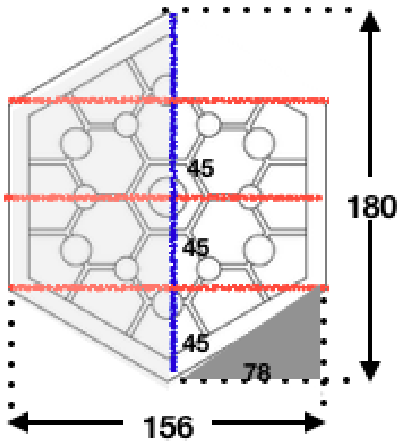
(An astute observer might realize that dividing by 2 is not necessary since there are two of these triangles at the top, and two more at the bottom, so they could actually multiply (45)(78) by 2! The student might also see that the four triangles could create a 156 x 45 rectangle with an area of (156)(45) = 7,020)

The area of all four triangles is (4)(1755) = 7,020.
The area of the large rectangle with the four triangles removed will give the area of the hexagon, 28,080 - 7,020 = 21,060 cm^2.



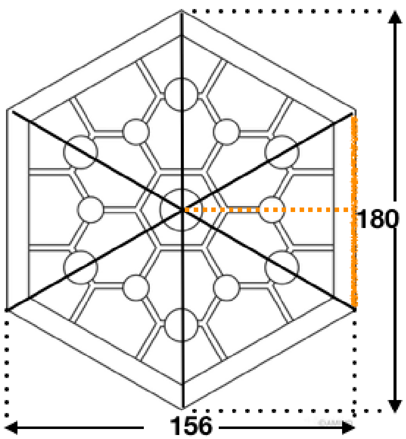
This student saw a large equilateral triangle created by three shorter diagonals. By dividing the vertical diagonal in fourths, it can be determined that each of the sections created is 180/4 = 45, and the height of the equilateral triangle is three of those sections, (3)(45)=135. The base of the equilateral triangle is the horizontal width given, 156, so the area of the equilateral triangle is (135)(156)/2 = 10,530. (This area could also be found by decomposing the equilateral triangle into two right triangles with a base of 78 and height of 135, then adding their areas together.)
What is left are three isosceles triangles with a base of 156 and height of 45. Students might find the area of one,
(45)(156)/2 = 3,510, then multiply by 3, (3,510)(3) = 10,530 (Note that the student might instead decompose the isosceles triangles to find the area of 6 right triangles with a base of 78 and height of 45.)
The area of the hexagon is the area of the equilateral triangle and the area of the three isosceles triangles combined, 10,530 + 10,530 = 21,060 cm^2.

Notice that the sum of the outer triangles is equivalent to the area of the large triangle! Perhaps a student might cut the three outer triangles off and into their halves and arrange them to show that they can combine to form the large triangle.



This last strategy shows a student who noticed that the hexagon is composed of two congruent isosceles trapezoids. In working with trapezoids earlier, the student knows that a triangle can be removed from one side of the trapezoid and arranged on the other side to form a rectangle. The orange lines drawn along those horizontal folds divide the diagonal of 180 into 4 equal sections of 45 cm each. When the triangle is moved to the other end of the trapezoid, the length of the rectangle formed is (3)(45) = 135.
The area of the rectangle is (135)(78) = 10,530
There are two trapezoids (now rectangles), so the area of the hexagon is (2)(10,530) = 21,060 cm^2.
How happy would you be if this student noticed the previous strategy from this drawing as well?!

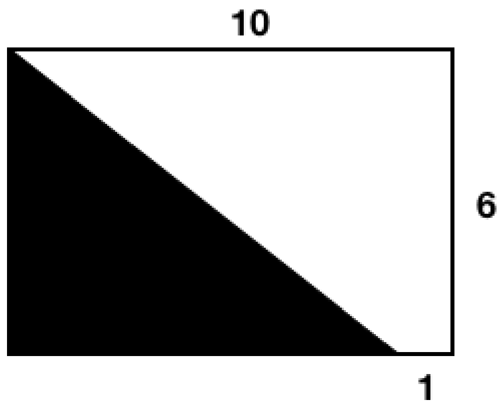
Student Misconceptions



- Students may simply multiply the dimensions given, (156)(180) = 28,440 cm^2
- Students may see the 6 triangles, but think that the height of each is 90 cm (the same as the side measurements), (90)(90)/2 = 4,050. Then (4,050)(6) = 24,300 cm^2
- Students may see half of the 156 dimension as the height of one of the 6 triangles and use 180 as the base of the triangle. They would multiply (180)(78)/2 = 7,110. Then multiply that by 6.
- (7,110)(6) = 42,660 cm^2

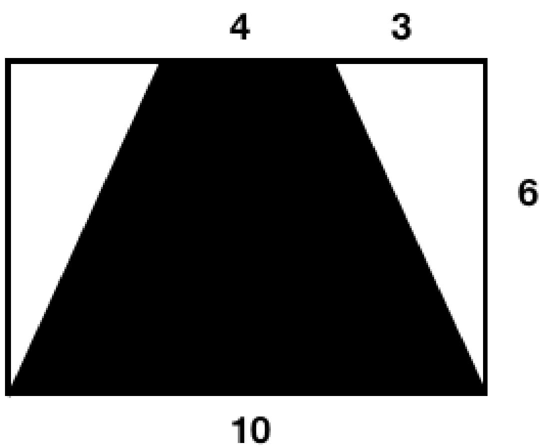
Anticipated Responses (Exit Ticket):

1.



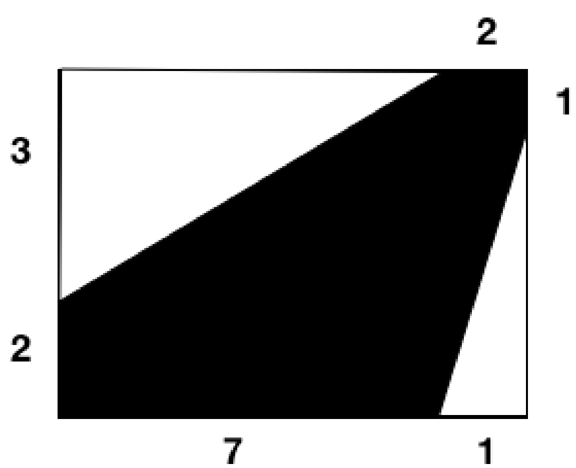
Area of shaded region: 27 square units
Because the figure is a rectangle, the shaded area is a right triangle. The height is 6 units.
The base would be 10 - 1 or 9 units.
(6)(9)/2 = 27
OR The triangle can be seen as half of a 6 by 9 rectangle.
Therefore the area is (6)(9)/2 = 27

2.



Area: 42 square units
The two unshaded triangles are congruent because the missing measure on the top of the rectangle must be 3, since 4 + 3 + 3 = 10, the measure of the opposite. The left side of the rectangle must have a measure of 6. The unshaded triangles on each end of the trapezoid have a base and height of 6 and 3, and can be combined to form a 6 by 3 rectangle. Its area would be (6)(3)=18 square units. The area of the whole rectangle is (6)(10), or 60 square units. The area of the shaded trapezoid can be found by subtracting the unshaded areas from the area of the whole rectangle. Therefore, the trapezoid has an area of
60 - 18 = 42 square units.

3.



Area: 29 square units
Since the large shaded portion is contained in a rectangle, opposite sides must be congruent. So the rectangle is 5 units by 8 units. The missing measure on the top then is 6 and on the right side is 4.
The area of the whole rectangle is (5)(8) = 40 square units.
The area of the larger triangle is half of a 6 by 3 rectangle,
9 square units.
The area of the smaller triangle is half of a 1 by 4 rectangle, 2 square units.
The area of the shaded trapezoid area be found by subtracting the unshaded areas from the area of the whole rectangle. Therefore, the area of the shaded region is
40 - 9 - 2 = 29 square units.