NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC²ML Research-Practice Brief #1

NC Math 1 – Equations & Introduction to Functions

BUILDING A STRONG FOUNDATION

The study of equations and functions in NC Math 1 builds extensively from students' prior experiences with algebra and function in elementary and middle school mathematics. The goals of this unit are for students to: 1) construct and solve expressions, equations, and inequalities from a given context, and 2) distinguish key features of a function given multiple representations. Specifically, students will: identify and interpret parts of algebraic expressions (A-SSE.1); create and solve equations and inequalities for a particular quantity or variable (A-CED.1&4, A-REI.3) while reasoning through their solution processes (A-REI.1); graphically represent solutions of linear equations and inequalities (A-REI.12); and evaluate and interpret functions based on their key features and representations (F-IF.1,2,&4). Further, this collection of standards progresses through NC Math 1, NC Math 2, and NC Math 3 and connects to all 4th level math course options.

ALGEBRA AND FUNCTION ACROSS K-8

Algebra and *Function* are combined domains in K-8, as students begin by expressing patterns and generalizations with variables and graphing ordered pairs. Students use variables to express mathematical relationships, and progress toward utilizing equations in which variables represent quantities that vary in relation to one another.

In 6th and 7th grades, pattern generalization continues from K-5 as students investigate ratios and proportions. In 8th grade, students are introduced to a formal definition of function, experience how functions describe relationships between quantities using words, variables, and graphs. Particular attention is given to linear relationships in the form $y=mx+b$.

Students' experience of connecting representations of mathematical relationships (verbal descriptions, numerical solutions, and formulas) is a continuation of their experiences that begin in 6th grade when students first build equations and inequalities using variables. Their understanding of algebraic notation and skill with symbolic manipulation will support their work with **functions**, since algebra is a tool for better understanding the relationships expressed by functions. Students also begin including the additional representation of graphs as they represent solutions of inequalitites as regions in the plane (NC.M1.REI.12) and connect the algebraic and graphical characteristics that define and demonstrate a function.

EXPRESSIONS, EQUATIONS, AND FUNCTIONS

As students are beginning their NC Math 1 experience, it is helpful to consider key findings from research on the ways students engage with algebraic **expressions**, **equations**, and **functions**. Expressions are a group of symbols that make a mathematical statement. Equations are mathematical sentences involving an equal sign. Parallel to grammar isntruction, a mathematical sentence must include a verb (e.g., "is equal to").

QUESTIONS TO CONSIDER WITH COLLEAGUES

What conceptions of equations do students exhibit as evidenced by their statements below?

$$\frac{1}{2}(x-4) = 10$$

- *"I would divide both sides of the equation by ½ because that's how you get rid of the ½."*
- *"Since the two sides are equal, multiply both sides by 2, then the left-hand side is simpler to solve."*
- *"If half of something is 10, then that something is 20. So x - 4 must equal 20."*

In contrast, the **function** concept is distinguished from "expression", "equation", and other relations. In high school, a function may be defined as

A relation that pairs every element in one set, called the domain, with exactly one element of a second set, called the range.

Always relating an input element to its only output element is the predictive, pattern-like power of functions that make them so useful within mathematics and beyond. Students engage with algebraic examples of functions (e.g. *f(x)=3x+7)* more often than other representations, therefore, they may conflate functions and equations (Ronau et al., 2014). Thus, it may be useful to first highlight non-algrebraic examples that focus on single-valued mapping from domain to range to support students in distinguishing between relationships that are functions and those that are not. For example, you could ask students (**domain**) questions about themselves (e.g. hair color, birth month, clothing color(s)), have them stand in respective places in the classroom (**range**), and consider which relationships are functions and why.

AN EXAMPLE: THE CUSTOMERS TASK

In the Customers Task (Illustrativemathematics.org) students are asked to determine a rule that describes the relationship between a set of customers and a set of home phone numbers.

Customer Name	Home Phone Number
Heather Baker	3105100091
Mike London	3105200256
Sue Green	3234132598
Bruce Swift	3234132598
Michelle Metz	2138061124

https://www.illustrativemathematics.org/content-standards/tasks/624

While this task focuses on F-IF.A.1, it also allows students to develop an understanding of the function concept while supporting the development of flexibility toward the types of elements that can define the domain and range.

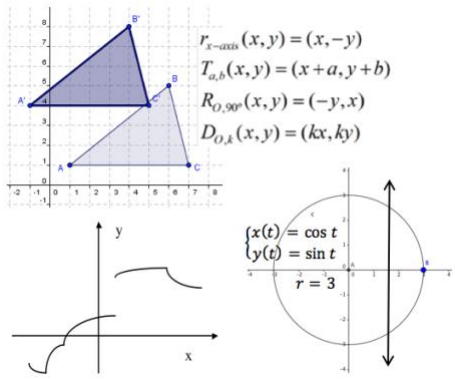
EXPANDING THE NOTION OF FUNCTION

While function notation is familiar to teachers and ubiquitous across mathematics curricula, it is not explicit in the standards until NC Math 1. The mathematical sentence "y = f(x)" shows that "y" and "f(x)" are the same value, naming a function "f", with a generalized input variable "x", and a function value or output "f(x)". This statement allows for substitution and builds on prior knowledge of y=mx *+b.*

An important aspect of the function concept is that functions apply to a wide range of situations, not all of which can be expressed using equations. It is important that students examine function and non-function relationships in different contexts that are not limited to a single variable domain as it lays an important foundation. For example, in NC Math 2 students extend their understanding of function to transformations as functions, with two-dimensional domain and range elements within NC.M2.F-IF (think **f(x,y)**, instead of **f(x)**). Here students view a collection of ordered pairs that create an object in the plane, a pre-image, as input values that can be entered into a function to produce the transformed, output image.

In addition, because students often engage heavily with linear functions, they may think that the range **must** map back onto the domain (Ronau et al., 2014). These types of functions represent a special case called a "one-to-one" function, where the inverse of the function is also a function. Furthermore, students may believe that functions must have numeric domains, be continuous, or pass the vertical line test. While the vertical line test may be initially helpful, if not explicitly discussed, students' partial understandings related to this misconception may persist. Shown in Figure 1 are counterexamples to these partial understandings (e.g. transformations, piecewise, & parametric respectively) that teachers draw upon to support students in building understanding of function.

Figure 1. Counterexamples to students' preconceptions



QUESTIONS TO CONSIDER WITH COLLEAGUES

- *How would you explain the concept of a function?*
- *What partial understandings do students sometimes have about functions?*
- *Where do these partial understandings come from?*
- *What examples or counterexamples can you introduce to confront these partial understandings?*

References

Ronau, R., Meyer, D., & Crites, T. (2014). *Putting Essential Understanding of Functions into practice in Grades 9-12*. B. Dougherty (Ed.). Reston, Va.: National Council of Teachers of Mathematics.

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NC²ML MATHEMATICS ONLINE

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SUGGESTED CITATION

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