NC Math 2- Square Root and Inverse Variation Functions

This unit in the HS Instructional Framework for NC Math 2 focuses on two different function types, square root functions and inverse variation functions. Building from the Transformations and Quadratic Functions units, this unit extends students' opportunities to interpret structure in expressions, create and solve equations while justifying their solution methods, build functions to solve systems of equations and inequalities, and interpret and analyze functions using multiple representations. The remaining standards unique to the Square Root and Inverse Variation Functions unit ask students to use these processes to extend the properties of exponents to rational exponents (NC.M2.N-RN.1,2), use properties of rational and irrational numbers (NC.M2.N-RN.3), solve inverse variation and square root equations and inequalities (NC.M2.A-REI.1,2,11), and apply this knowledge to inverse and square root functions.

BUILDING FROM MIDDLE GRADES

Students begin to engage formally with exponents in 6th grade when they use whole number exponents to represent powers of 10 when writing and evaluating numerical expressions (NC.6.EE.1). Building from this, students in 8th grade develop and apply properties of integer exponents to generate equivalent expressions (NC.8.EE.1), formalizing that $x^{-n} = 1/x^n$. Students in 8th grade also explore expressions involving square and cube roots and evaluate radical values of perfect squares or cubes (NC.8.EE.2), and use rational approximations for radicals to compare "non-perfect" real numbers (NC.8.NS.2). Formalizing radicals to rational exponents is not done in 8th grade, but is reserved for students in NC Math 1 (NC.M1.N-RN.2), where students rewrite algebraic expressions with integer exponents using the properties of exponents.

PATTERNS OF EXPONENTS: MULTIPLICATION & DIVISION

All learning happens when connections are made from existing knowledge to new knowledge. The rules of multiplying or dividing powers with like bases and raising a base to an exponent can be seen through *generalizing* examples or *examining* patterns that build from students' prior knowledge. Similarly, the notation for writing radicals as exponents can also be developed using these processes. NC Math 2 students will need support in drawing upon their conceptions of where these "rules" come from and why they work to formalize writing radicals as exponents. The *Evaluating a Special Exponential Expression* task from illustrativemathematics.org helps students build conceptual understanding of these rules.

RECOGNIZING PATTERNS

One possible way to develop student conceptual understanding of exponent "rules" is to give students a string of expressions and have them look for patterns. For example,

- $10^8/10^8$; $1.5^4/1.5^4$; $9^{½}/9^{½}$
  Why does any non-zero base raised to the zero exponent equal 1?
- $\sqrt[3]{27} = \sqrt[3]{3^3} = 3^{\frac{3}{3}} = 3^1 = 3;\ \sqrt{3} = \sqrt{3^1} = 3^{\frac{1}{2}}$
  How do you relate the notation for radicals and rational exponents?
- $3^3 = 27; 3^2 = 9; 3^1 = 3; 3^0 = 1; 3^{-1} = {}^1/_3$
  How do you develop the meaning behind negative exponents?

INVERSE VARIATION VS. DIRECT VARIATION

The mathematical relationship of variables that exist in *direct variation* to one another is a major focus of middle grades mathematics. Whenever quantities represented by **x** and **y** exist in direct variation to one another, then **y = kx**, for some non-zero real number **k**. Direct variation occurs when two quantities are proportional, since **y/x** will always equal the value **k**. Though we spend a large portion of our 6-8 standard course of study devoted to this type of mathematical relation, students may still have partial conceptions of direct varation. In addition, students may think that linear functions vary directly (e.g. are proportional) simply because they decrease or increase at a constant rate (Bush & Karp, 2013).

Inverse variation is a mathematical relationship between quantities (again as **x** and **y**), in which **y = k/x** or as its often expressed **xy = k** for some non-zero real number **k**. Within *direct variation* if **y** increased, then **x** would also have to increase so that their ratio would always equal **k**. With inverse variation, if **y** increased then **x** would have to decrease so that their product would always equal **k**. In a study of MS students' preconceptions prior to formal instruction, researchers found that students had more difficulty with verbally and symbolically describing inverse variation problems in relation to other problems (e.g. direct variation, linear, exponential) (Swafford & Langrall, 2000).

The HS Instructional Framework speaks to these connections between direct and inverse variation. Though inverse variation is the focus of NC Math 2, it is important to make the connection between the two types of variation. Examining the relationships between "similar" direct variation and inverse variation (e.g. y=5x and y=5/x) using multiple representations supports students understanding inverse variation and building a stronger understanding of direct variation.

ENGAGING WITH MULTIPLE REPRESENTATIONS

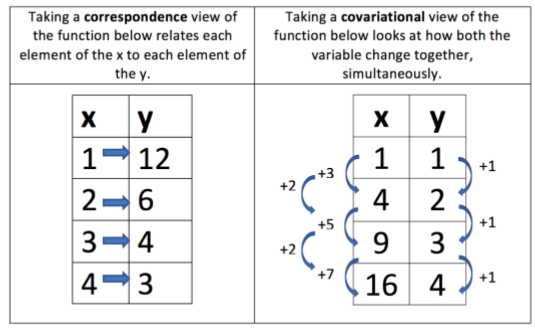
Ronau et al. (2014) highlight that it is important for students to engage in mathematics in ways that build toward a strong conceptual understanding of functions. Namely, that different representations highlight different characteristics of a function; some representations may be more useful than others; and links between algebraic and graphical representations of functions are especially important in studying relationships and change.

Examining graphical or tabular representations may bring forth the inverse relationship between quadratic and square root functions that highlights the similarities and differences between their rates of change. Connecting the idea that the inverse of a quadratic function, namely a square root function, has x-values that change linearly, similar to the y-values in a quadratic function, sets the foundation for defining inverse relationships. While graphs of quadratic functions have symmetry, students may recognize that graphs of square root functions do not and wonder why that is the case. This provides an opportunity to connect the *Algebra* and *Function* domains.

STUDENTS' THINKING ON FUNCTIONS

As previously mentioned, quadratic functions preceeds this unit on square root and inverse variation functions. While quadratics present a great opportunity to point out the linear rate of change of quadratic functions, this unit also presents a great context to make both correspondence and covariational perspectives explicit.

Correspondence is the horizontal relationship found in a traditional function table. It defines functions abstractly and focuses on the rule that relates each input to its output. Correspondence describes how to find y from a given x. On the other hand, ***covariation*** is the vertical relationship, describing changes in the dependent variable that happen concurrently with changes in the independent variable. Students intuitively build functions using a covariational approach. In this fashion, they are able to describe changes in the data columns and even interpolate within the data set (Confrey & Smith, 1995). Using a covariational approach to teach functions can lead to a more in-depth understanding and a better sense of the relationships that functions define. When looking for square root patterns, a covariational perspective will help students see how the two variables are simultaneously changing. However, students should not ignore the relationship between inputs and outputs that a correspondence perspective highlights. In fact, students will need to have a correspondence perspective to identify that **xy = k**.



The table on the left represents the relationship $xy = 12$ *and the table on the right,* $y = \sqrt{x}$.

QUESTIONS TO CONSIDER WITH COLLEAGUES

- How can you draw upon what students learned in previous units or courses to support them in understanding square root and inverse variation functions?
- How can you increase opportunities for students to make connections between different representations?

REFERENCES

Bush, S. B., & Karp, K. S. (2013). Prerequisite algebra skills and associated misconceptions of middle grade students: A review. *The Journal of Mathematical Behavior*, *32*(3), 613-632.

Confrey, J., & Smith, E. (1995). Splitting, covariation and their role in the development of exponential functions. *Journal for Research in Mathematics Education,* 26, 66.86.

Ronau, R., Meyer, D., & Crites, T. (2014). *Putting Essential Understanding of Functions into practice in Grades 9-12*. B. Dougherty (Ed.). Reston, Va.: National Council of Teachers of Mathematics.

Swafford, J. O., & Langrall, C. W. (2000). Grade 6 students' preinstructional use of equations to describe and represent problem situations. *Journal for Research in Mathematics Education*, 89-112.

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SUGGESTED CITATION

NC²ML (2018, October). NCM2.4 Square Root and Inverse Variation Functions. *Research-Practice Briefs*. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. Retrieved from nc2ml.org/brief-10