NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC^{2}ML Research-Practice Brief #12

NC Math 2 – Probability

CONNECTING TO 6-8 MATHEMATICS

A prominent goal of the probability standards for NC Math 2 is for students to understand, explain, and use conditional probabilities. This includes understanding when and how to apply the addition and multiplication rules for probability.

There is a likelihood that NC Math 2 students have not worked with probability since 7th grade, where essentially all of the middle grades probability standards are clustered. Even so, students should be familiar with the definition for the ***probability of an event*** (i.e., *a measure of the likelihood that the event will occur)* as it is central to achieving the standards in 7th grade. The less familiar interpretation of the ***probability of an event*** is to understand it *as a long-run relative frequency.* This interpretation is a part of probability in 7th grade.

> ***NC.7.SP.6*** *Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.*

The cluster of 7.SP-standards involve creating probability models to run simulations of events and record the observed frequencies (NC.7.SP.8.c). These simulations are the "chance processes" of NC.7.SP.6, that produce the ***experimental probabilities*** calculated by *the ratio of relative frequency to the total number of simulations*. 7th grade work with proportions will help students be able to approximate the relative frequency given the (theoretical or observed) probability. See the North Carolina 7th Grade Standards for more details.

RANDOMENESS: 7TH GRADE TO MATH 3

After the 7th grade experience with probability, 8th grade statistics includes the creation and interpretation of two-way tables, a representation that is a powerful tool for identifying potential association between two variables and for making sense of conditional probability. NC Math 1 statistics has a major focus on identifying potential relationships between two variables (linear regression), while separating association from causation. The simulation design in 7th grade and NC Math 2 that ensures **randomness in sampling** becomes even more important in NC Math 3 when students learn about **random sampling** techniques that are employed to gain a **random sample** from a population. An ideal random sampling technique is one for which every member of a population is equally likely to be chosen in the sample.

The 7th grade probability cluster extends to NC Math 2 as students again use simulations to explore probability. One distinction between the 7th grade cluster and the NC Math 2 standard NC.M2.S-IC.2 is the notion of ***sample data*** that is representative of a ***population***. Randomness in sampling is vital to prevent bias, and truly represent the entire population.

STUDENTS' CONCEPTIONS OF RANDOMNESS

So what do we know about how students think about randomness? Research says that students sometimes have a *representativeness* view of randomness (Garfield, 1995). They expect all samples to closely "represent" the population. For example, consider the probability of getting heads when flipping a coin. Students may believe that having seen a coin land on heads 80% of the time for 10 flips, then it would be equally likely for 1000 flips. Also, with a representativeness view students expect that the probability of getting a heads after landing on tails 4 times in a row is more likely than the outcome of tails. This is commonly referred to as Gambler's Fallacy (Jones & Thornton, 2005). Students are not wrong in realizing that 5 tails in a row are highly unlikely, however, they should also understand that coin flips are independent events. The conflict is distinguishing between the independence of 5 single coin flips versus the probability of the string of 5 flips. Konold et al. (1993) suggests engaging students in a conversation about the difference of these two situations.

AN EXAMPLE: AGREE/DISAGREE

Think about the following situation that you could pose to students:

> *Carlos flips a coin 4 times and gets T, T, T, T. He claims that the probability of getting a T on the next flip is 50/50. Barb disagrees, and says that the probability will be much less since landing on "T" five times in a row is very slim. Who do you agree with? Why?*

Both students are correct, however they are answering different problems. Carlos is answering the probability of a coin landing on tails. Barb is answering the probability on landing on five tails in a row. Introducing this cognitive conflict creates opportunities for classroom discussion to further develop students' understanding of randomness and probability.

TOOLS FOR UNDERSTANDING PROBABILITY

The HS Instructional Framework discusses the importance of students physically conducting simulations using tactile tools such as spinners, dice, or colored tiles in a bag. Using technology allows students to quickly run large-scale simulations and can be helpful in fostering discussions comparing experimental and theoretical probability, thus allowing students to make sense of the connections between the two (Stohl, 2005). Consider a basketball player with a 60% shooting percentage. Using a weighted spinner (60% make/40% miss), students can simulate the number of shots made over the next 10 free throws, then compare their experimental results with the theoretical. Many students take these tools (like the spinner) at face value, not realizing they can be used to represent different situations. Another example of using a tool to represent something more meaningful is flipping a coin to represent the birth of a boy or girl. Check out the CPM Probability Tools if you don't have any of these hands-on tools for your classroom. There's also an adjustable spinner at NCTM's Illuminations website.

These tools can support students in understanding that **outcomes** of a chance process are the elements of the **sample space** of all possible outcomes, and that **events** are subsets of the sample space. Understanding these relationships as set and subset relationships is vital to meet NC.M2.S-CP.1, in which students describe events as results of the set operations of unions, intersections, and complements. Actually simulating an experiment involving "without replacement" (e.g. marbles in a bag) allows students to understand how "not replacing" affects the probability of the next outcome. Research has shown that tasks that involve "without replacement" can be more challenging for students because they often fail to attend to the changes in sample space (Tarr & Lannin, 2005).

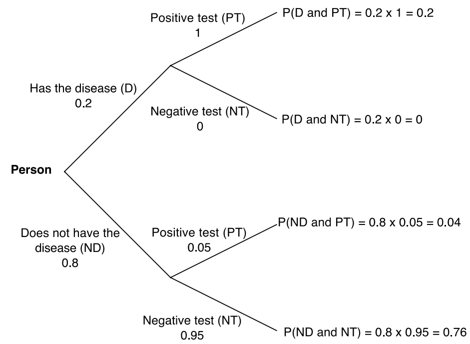
AN EXAMPLE: SURVIVING THE TITANIC

Conditional probabilities can be understood very naturally through contexts and displays of data. Opportunities to connect context and representations will support standards NC.M2.S-CP.3-5 where the goals are for students to *understand, recognize, and explain the concepts* of **conditional probabilities** and **independence of events.** Consider this data on passengers of the Titanic where survival is more likely for some than it is for others. Using the two-way table, consider the following 4 questions:

Passenger data from- www.icyousee.org/titanic.html	Men	Women	**Total**
Survived	128	304	**432**
Died	648	108	**756**
Total	**776**	**412**	**1188**

1. *Calculate the probability of a passenger surviving the wreck.*
2. *What's the probability of survival given that you are a female passenger?*
3. *What's the probability of survival given that you are a male passenger?*
4. *Why might those probabilities be different?*

Another very helpful representation to use when considering probabilities is the tree diagram. Tree diagrams provide an organized view of the probabilities of events in a sample space, like this one from the *Mathematics Assessment Project's Medical Testing*. Consistent use of tree diagrams with simple and complex examples, supports understanding of the Addition and Multiplication rules of probability in NC.M2.S-CP7&8.



QUESTIONS TO CONSIDER WITH COLLEAGUES

- *Reflecting on the Titanic and Medical Testing Tasks, what do the different representations show/hide?*
- *How do they represent the sample spaces and events?*
- *How can we support students making connections between these representations of data and the context of the data?*
- *What connections do you see between the statistics in NC Math 1, 3 and the probability in this unit of study?*

References

Garfield, J. (1995). How students learn statistics. *International Statistical Review, 63(1), 25-34.*

Jones, G. A., & Thornton, C. A. (2005). An overview of research into the teaching and learning of probability. *In Exploring Probability in School (pp. 65-92).* Springer US.

Konold, C., Pollatsek, A., Well, A., Lohmeier, J., & Lipson, A. (1993). Inconsistencies in students' reasoning about probability. *Journal for Research in Mathematics Education, 24(5), 392-414.*

Pratt, D. (2005). How do teachers foster students' understanding of probability?. *Exploring Probability in School, 171-189.*

Stohl, H. (2005). Probability in teacher education and development. *Exploring Probability in School, 345-366.*

Tarr, J. E., & Lannin, J. K. (2005). How can teachers build notions of conditional probability and independence?. *In Exploring Probability in School (pp. 215-238).* Springer US.

LEARN MORE

Join us as we journey together to support teachers and leaders in implementing mathematics instruction that meets needs of North Carolina students.

NC²ML MATHEMATICS ONLINE

For more information on accessing Canvas learning modules or additional resources please visit http://nc2ml.org/

SUGGESTED CITATION

NC²ML (2018, October). NCM2.6 Probability. *Research-Practice Briefs*. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. Retrieved from nc2ml.org/brief-12