NC Math 3 – Exponential and Logarithmic Functions

LOOKING ACROSS NC MATH 1, 2, AND 3

Exponential functions are not new to students, since they are a topic of study in NC Math 1. In NC Math 1, students engage with exponential functions, building an understanding of multiplicative rates of change. That is, exponential functions can be characterized by a rate of change that is proportional to the value of the function.

In the NC Math 3 **Exponential and Logarithmic Functions Unit** students are blending the domains of *Algebra* and *Function*. Being able to create algebraic equivalencies will support students' abilities to understand and interpret exponential and logarithmic relations. Students will also connect the verbal descriptions to the symbolic and graphical representations to these new function types. This work of blending these representations in the A-CED, A-SSE, and F-IF standards is common across all three math courses in NC.

MOTIVATING EXPONENTIAL FUNCTIONS

Populations grow or decay based on the number of members originally present in the population. The amount of compound interest one earns depends on the original amount of money in the account. Carbon dating is completed by the process of determining an isotope's half-life, or how long before half of the amount of the isotope has decayed. Here the amount of decay will depend upon the amount that's initially present. All of these applications share the characteristic that the rate of change in the quantities are dependent upon the amount of the quantities.

Exponential functions are broadly used to model these and other phenomena over time. Though NC Math 3 students are not required to explore exponentials with base *e*, they will be studying examples with rational number bases, to distinguish them from polynomial functions by their end behavior.

MOTIVATING LOGARITHMS

Logarithms were the tools of mathematicians, engineers, and scientists, used for quick calculations before the widespread use of calculators and computers. Numbers, even very large or very small numbers, can be multiplied and divided more quickly by calculations with their equivalent logarithm values (check out the Numberphile example here. Based on the video think about how you can build on students' prior knowledge of exponent rules from 8th grade and NC Math 1 to introduce logarithms.)

Today, their applications allow for scaling down or compressing large quantities into smaller, more manageable values and modeling phenomena that grow exponentially with a linear model (increasing by a factor of 10 corresponds to an additive increase of 1 to the logarithm). It also helps to problematize the reason for counting in other bases, as is true in computer science. For a more artistic view of logarithms, try Vi Hart's video.

Following their learning of inverse functions, NC Math 3 students can be introduced to the logarithmic function (base 10 – the *log* function on your calculator) as the function that is the inverse of the exponential function $y = 10^x$. It will be useful for students to experience the general logarithmic relation $y = \log_b x$, since this relationship is used to solve for a variable in an exponent. NC Math 3 students will not have to engage with all the properties of logarithmic relations (see the HS Instructional Framework for further clarification of standard F-LE.4), so that time can be spent instead helping students make sense of the function and its notation.

BUILDING POWERFUL MEANINGS FOR LOGARITHMS

Given the inverse relationship between exponents and logarithms, it is important to take care with the way we connect them for students. Research has shown that since logarithmic equations are introduced in connection to exponential equations, students sometimes think of them as one and the same (Kastberg, 2002; Siebert, 2017). This is often referred to as developing a ***conversion meaning*** of a logarithm. When students are told that $log_b a = c$ is equivalent to $a = b^c$ and they practice this conversion repeatedly, they come to see this conversion process as the meaning of a logarithm (Siebert, 2017). This is problematic because it undermines sensemaking about what a logarithm actually is: a function. While learning to convert is necessary, it is important to build a meaningful understanding of logarithm first so that the misconception of logarithms as a conversion does not impede further learning. Consider the suggestions in the box below as you engage students in developing powerful meanings of logarithms.

When moving back and forth between talking about logarithmic expressions and equations and logartihmic functions, it is important to attend to your students' understanding of the notation. For example, Hurwitz (1999) notes that "students often have difficulty thinking of a logarthims as the output of a function because the notation used for logarithms does not look like the familiar f(x) notation" (p. 334). In other words, with polynomial functions (e.g., $f(x) = 2x^3 + 3x$) it is clear what actions should be taken, but for a logarithmic function (e.g., $f(x) = log_2(3x)$ that action is not clear. Furthermore, as Weber (2002) points out in all logarithmic functions of the form $log_a x = y$, the input value, x, can be thought of as both a domain value for the logarthmic function and the product of *y* factors of *a*. So, listen carefully to your students as they share their thinking when working with logarithmic equations and functions so that you pick up early on any misconceptions like those noted here.

AN EXAMPLE: LOG LATTICE

The following task is taken from the website NRICH Enriching Mathematics and is a great one for building powerful meaning of logartithms. Use some of the following logarithms to complete the table. The values of the logarithms need to increase along the rows and down the columns. Do this without calculating by approximating values for the expressions.

$\log_3 2$ $\log_4 5$ $\log_2 5$ $\log_3 4$ $\log_3 5$ $\log_5 3$

$\log_4 2$ $\log_2 4$ $\log_2 3$ $\log_5 2$ $\log_5 4$ $\log_4 3$

	Increasing size ⟶		
Increasing size ↓	$\log_\square \square$	$\log_\square \square$	$\log_\square \square$
	$\log_\square \square$	1	$\log_\square \square$
	$\log_\square \square$	$\log_\square \square$	$\log_\square \square$

Reflect on your strategies:

- Can you put pairs of logarithms from the list in order of size? If so, how?
- Which logarithms are bigger than one and which are smaller? How do you know?

MAKING SENSE OF LOGARITHMS

Think of "**log**" as "**the power you raise 10 to**" so that **log 100** is "**the power you raise 10 to, to get 100**" and so **log 100 = 2**, since "**the power you raise 10 to, to get 100 is 2".** Then "**$\log_2 16$**" is "**the power you raise 2 to, to get 16**" or **$\log_2 16$ = 4**. You may decide to get to the logarithmic property your students need by building on this translation and examining patterns.

$$2 \cdot \log 10 = 2 \cdot 1 = 2 = \log 10^x?$$
$$3 \cdot \log 10 = 3 \cdot 1 = 3 = \log 10^x?$$
$$2 \cdot \log 100 = 2 \cdot 2 = 4 = \log 100^x?$$
$$4 \cdot \log \frac{1}{10} = 4 \cdot -1 = -4 = \log \frac{1}{10}^x?$$

QUESTIONS TO CONSIDER WITH COLLEAGUES

- *Given each of these images involving exponentials and logarithms, what do you notice? What do you wonder?*
- *How does the image assist you in making sense of exponentials, power functions and logarithms?*

tinyurl.com/expsandlogs (Vanderwerf, 2016)

References

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