NC^{2}ML Research-Practice Brief #2

NC Math 1 – Linear Functions

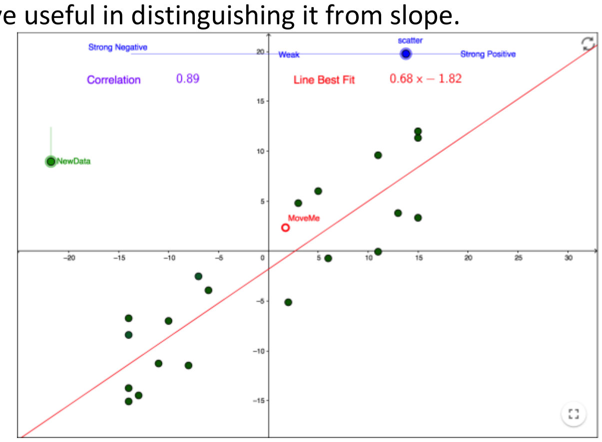
NC MATH 1 - LINEAR FUNCTIONS UNIT

The second unit of the HS Instructional Framework, the *Linear Function* unit, follows seamlessly from the *Equations and Introduction to Function* unit and prior, related K-8 mathematics. Keeping the connection between *Algebra* and *Function*, in this unit students are expected to recognize the graphical meaning of parameters **a** and **b** within the equation **f(x)=ax+b** (F-LE.5) and be able to manipulate the variables, justifying their algebraic methods (A-REI.1). Additionally, students will generate linear equations to model situations (A-CED.1, 2), identifying the meaning of coefficients and variables within a context (A-SSE.1), and understand that the graph of the equation shows all possible domain and range pairs (F-IF.5) that are solutions to the equation (A-REI.10).

CONNECTIONS TO GEOMETRY & STATISTICS

This unit includes applications of linear relationships to geometry and statistics. The geometry connection in this unit calls for the comparison of **slope** values when proving lines are **parallel**, **perpendicular**, or neither (G-GPE.5). Having learned how to calculate the distance between two points in the coordinate plane in 8th grade, students will calculate midpoints of lines, the perimeters of planar figures, and justify the classification of triangles (G-GPE.4,6).

Statistical applications of linear relationships include fitting a **least squares regression line** to a scatterplot and using the relationship as a model for prediction (S-ID.6-9). This notion involves a deeper look at data using statistics in order to determine if a linear model is appropriate and if a particular linear model is a good fit to the data. A scatter plot and potential linear model can be examined visually to begin to predict how well the line fits the data. From there, the **correlation coeffcient** is the statistic used for measuring the strength of fit for a given line. It is important to highlight that examples exist of data with fitted lines having a high correlation coefficient, but for which the data is non-linear. Looking for patterns in **residual plots** will support students in making these determinations and in distinguishing between association and causation (S-ID.9). As students begin to interact with this new statistic, the correlation coefficient (r), the following applet from Geogebra.org may prove useful in distinguishing it from slope.



RESEARCH ON REPRESENTATIONS & SLOPE

Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and tables. Research has shown that many students compartmentalize their understandings of these representations and have difficulty coordinating among them (Ferrini-Mundy & Graham, 1994). Varying the way that a function is represented supports students in building both flexibility across representations and a broader understanding of functions.

While linear functions are characterized by a constant rate of change, students may hold narrow conceptions of slope that limit their understanding of linear functions, such as viewing slope only as a ratio or calculation, or as “steepness” rather than a value (Stump, 2001). Reasoning about the similarity of “slope triangles” may support students in building an understanding of linear functions as having a constant rate of change.

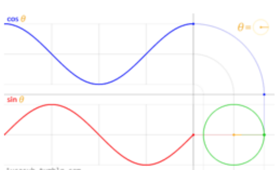
RESEARCH ON RATE OF CHANGE & COVARIATION

For some functions, patterns of behavior in their output categorize them into particular function families. Building from students' prior understanding of rate, ratio, and proportion in K-8, students in NC Math 1 are first exposed to linear functions – functions that have a **constant addititive rate of change**. That is, over equal intervals of input, the output changes by the same amount. Following this, students engage with quadratic and exponential functions as families of functions that do not have a constant additive rate of change.

To understand the behavior of a function and its' function family, teachers often support students by developing a rule of correspondence between input and output by focusing on the action of plugging in numbers and analzying the output. This approach is called a *correspondence* perspective of function. While useful, research has shown that by also attending to a *covariation* perspective of function, students can develop a stronger and dynamic understanding of rate of change across function families (Carlson et al., 2002).

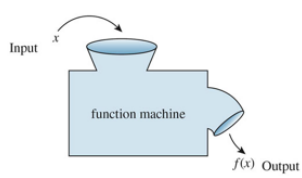
Covariation

- Logic of Learners: functions are the coordination of two quantities varying simultaneously (***covariation***)
- Dynamic and informal
- Mathematics of change



Correspondence

- Logic of Mathematics: functions are a special kind of ***correspondence*** between two sets
- Static and formal
- Mathematics of structure



A covariation view of function focuses on coordinating the change in two varying input and output values as one moves through a table or graph. This view makes the concept of rate of change explicit by highlighting repeatability in the input and output. Instruction and tasks that draw on this perspective engender student's prior understanding of rate and supports them in making sense of the uniqueness of individual function families. In addition, this approach has been found to support conceptual understanding of inverse functions, function compositions, and calculus (Oehrtman et al., 2008), which students engage with in NC Math 2, NC Math 3, and undergraduate mathematics. As a result, it is important to be intentional in providing experiences for students to explore covariation.

Covariation

Every hour, the distance goes up by 60

Table

Δx	Hours	Distance	Δy
	1	60	
+1	2	120	+60
+1	3	180	+60
+1	4	240	+60

Correspondence

Multiply the number of hours by 60 to find the distance

Hours	Distance
1 →	60
2 →	120
3 →	180
4 →	240

SEQUENCES FROM A FUNCTION PERSPECTIVE

Sequences are functions with domains that are very much like the set of natural numbers. The standard notation of sequence terms, $a_1, a_2, a_3, \ldots, a_n, \ldots$, uses the domain or input value as an index, however, standard function notation, ***a(1)***, ***a(2)***, ***a(3)***, …, ***a(n)***,…, may also help us remember that sequences are functions. In this unit students will build arithmetic (linear) sequence expressions from descriptions, graphs, and tables of values (F-BF.1a). Understanding patterns in both explicit and recursive forms will prepare them for revisiting sequences in Unit 4, when they will study geometric sequences (exponential).

QUESTIONS TO CONSIDER WITH COLLEAGUES

- *What conceptions do your students commonly have about linear functions?*
- *How could a covariational perspective support your students in building understanding of functions?*
- *How can you design appropriate interventions or tasks to advance or refine their conceptions?*
- *Given that sequences are functions, what are some limitations of the vertical line test?*

References

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education*, 352-378.

Ferrini-Mundy, J., & Graham, K. (1994). Research in calculus learning: Understanding of limits, derivatives, and integrals. *MAA notes*, 31-46.

Oehrtman, M., Carlson, M., & Thompson, P. W. (2008). Foundational reasoning abilities that promote coherence in students' function understanding. *Making the connection: Research and teaching in undergraduate mathematics education*, 27-41.

Stump, S. L. (2001). Developing preservice teachers' pedagogical content knowledge of slope. *The Journal of Mathematical Behavior*, *20*(2), 207-227.

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NC^{2}ML MATHEMATICS ONLINE

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SUGGESTED CITATION

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