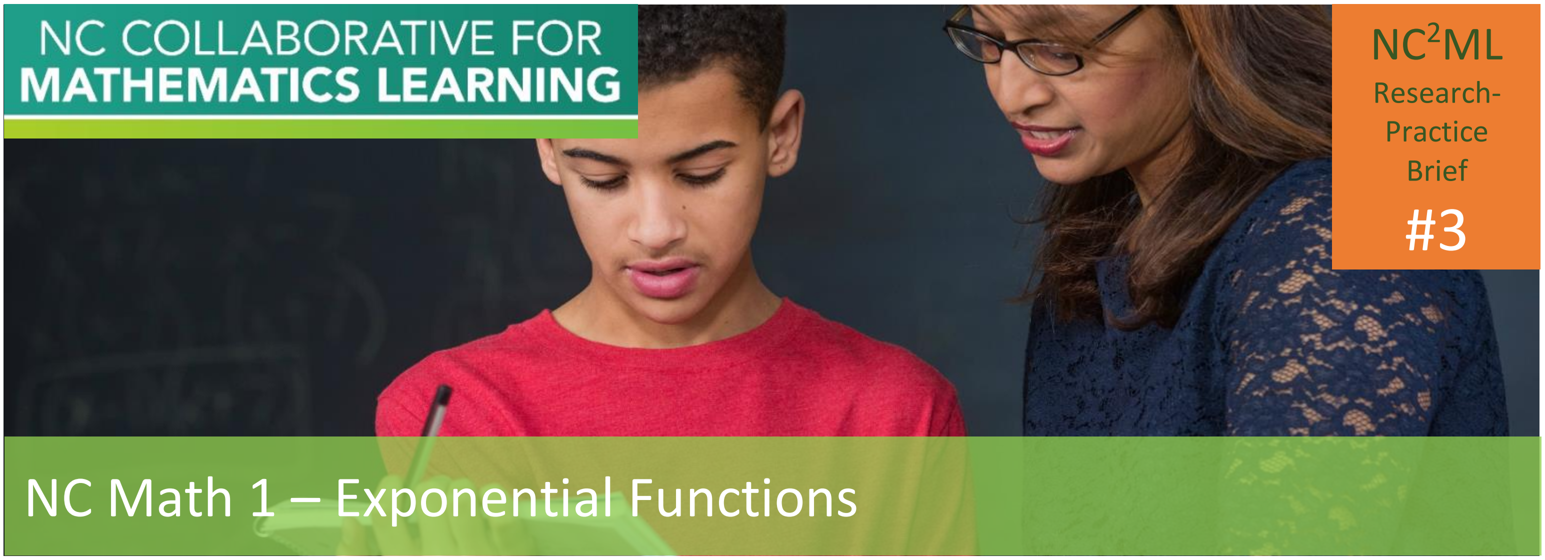

NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC^{2}ML Research-Practice Brief #3

NC Math 1 – Exponential Functions

NC MATH 1 – EXPONENTIAL FUNCTIONS UNIT

The ***Exponential Functions*** unit, flows well from the previous unit on *Linear Functions*. In this unit students learn to recognize exponential functions defined by $f(x) = ab^x$, where $b > 0$, a is the y-intercept, and b is the rate of growth or decay of the function. Students are expected to understand and apply exponent properties (NC.M1.N-RN.2). They will also work with geometric sequences, determining their explicit and recursive formulas (NC.M1.F-BF.2) and connecting them with exponential functions (NC.M1.F-IF.3).

This unit is also the first time students dig deeply into functions that do not have a constant rate of change. Students will evaluate, create, and interpret exponential functions in context (NC.M1.F-IF.2&4, NC.M1.A-CED.1&2, NC.M1.A.REI.10, NC.M1.F-IF.6, NC.M1.F-LE.5). They will not only compare, interpret, and explain key features of exponential functions, but also write and apply exponential functions given multiple representations (NC.M1.F-LE.1&3, NC.M1.F-IF.5,7, 8b & 9, NC.M1.F-LE.3, NC.M1.A-SSE.1, NC.M1.A-REI.11, NC.M1.F-BF.1, NC.M1.S-ID.6c).

BUILDING FROM MIDDLE GRADES

The use of exponents first appears in 6th grade when students use whole number exponents to represent powers of 10. Without additional exponent content added in 7th grade, 8th graders begin to generalize properties of integer exponents, thus understanding $x^{-n} = \frac{1}{x^n}$. 8th graders also explore expressions involving square and cube roots, evaluating the radical values of perfect squares or perfect cubes, and using multiplicative reasoning to determine decimal approximations for radicals of "non-perfect" real numbers. They do not formalize radicals to rational exponents. This is reserved for students in NC Math 2. However, any integer exponent is fair game in NC Math 1: (NC.M1.N-RN.2) *Rewrite algebraic expressions with integer exponents using the properties of exponents*.

CONNECTING RULES OF EXPONENTS

Research has shown that students' conceptions of exponents are critical in how they interpret exponential functions (Weber, 2002). Students must develop a true understanding of ***a^m*** as the product of ***m*** factors of ***a*** so that they will be able to generalize this rule when working with the laws of exponents, eventually rational exponents, and what that means for the nature of exponential functions (Weber, 2002). The rules for exponents can be seen through generalizing examples or examining patterns that build from students' prior knowledge. It is important for NC Math 1 students to truly understand where these "rules" come from and why they work, to prepare them for working with rational exponents they will encounter in NC Math 2.

QUESTIONS TO CONSIDER WITH COLLEAGUES

How do you support students in building understanding of the following?

- ***Why any non-zero base raised to the zero exponent equals 1***
  e.g. examining patterns in $\frac{2^3}{2^3}; \frac{10^8}{10^8}; \frac{1.5^4}{1.5^4}; \frac{9^{-4}}{9^{-4}} \ldots$
- ***Negative exponents***
  e.g. examining: $3^3 = 27;\ 3^2 = 9;\ 3^1 = 3;\ 3^0 = 1;\ 3^{-1} = \frac{1}{3}; \ldots$
- ***Dividing powers with like bases***
  e.g. examining: $\frac{3^5}{3^2} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3}$ or $3^{5-2} = 3^3$; $\frac{a}{a^3} = \frac{a}{a \cdot a \cdot a} = \frac{1}{a \cdot a}$ or $a^{1-3} = a^{-2}$

EXPONENTIAL FUNCTIONS

As NC Math 1 students working in Unit 2 (Linear Functions) add *having a constant rate of change* to their list of characteristics of functions that must be *linear*, it may be timely for a teacher to choose exponential and quadratic functions as non-examples of linear functions.

Choosing exponential and quadratic non-examples would be timely during the previous unit on *Linear Functions (Unit*

2), since **exponential functions** are the topic of Unit 3, and **quadratics** fill Unit 4. Just as linear functions can be characterized by a rate, so too are exponential functions, but not in the same way. A classic task that addresses these issues is The Summer Job task.

AN EXAMPLE: THE SUMMER JOB TASK

Three different people want to hire you to work for 12 weeks in the summer, each offering a different weekly salary. The 1st boss will pay you \$100 times the number of the week of employment (so week 1-\$100, week 2-\$200, etc.). The 2nd boss will pay you \$10 times the square of the number of the week of employment (so week 1-\$10, week 2-\$40, etc.). The 3rd boss offers you \$2 for the first week, but she will double your pay each week (so week 1-\$2, week 2-\$4, etc.). Assuming that the job is the same for each boss, which boss will you work for and why?

RESEARCH ON STUDENT THINKING ABOUT EXPONENTIAL FUNCTIONS AND RELATIONSHIPS

Research has shown that rate of change is a "primary point of entry" for students learning about exponential functions (Confrey & Smith, 1994, p. 34). Unlike linear functions students have previously encountered, exponential functions do not have a constant rate of change. Instead, the rate of change of an exponential function is not constant. However, there is a pattern in the way function values grow. Consider the table of values for the function $f(x) = 2(3)^x$.

x	f(x)	Δf(x)
1	6	
2	18	$18 - 6 = 12$
3	54	$54 - 18 = 36$
4	162	$162 - 54 = 108$

By calculating the ratios of consecutive funciton values students can identify the rate of growth. Confrey and Smith (1994) refer to this as a ***multiplicative rate of change***. In the example above, focusing students on attending to the ratio of consectuive y-values of an exponential function, students can determine that there is a constant ratio or growth factor of 3. In addition, by attending to this ratio students can also see that the rate of change is increasing at an increasing rate as each consecutive function value is tripled.

x	f(x)	$\frac{f(x+1)}{f(x)}$
1	6	
2	18	$\frac{18}{6} = 3$
3	54	$\frac{54}{18} = 3$
4	162	$\frac{162}{54} = 3$

RATE OF CHANGE VS. RATE OF GROWTH

It's important that teachers carefully distinguish for students "*rate of change*" from "*growth rate*" (or "*decay rate*"). The challenge to vocabulary may come when students must identify a growth rate and an average rate of change for an exponential function. A *growth rate* is a ratio of the amount of change in function values (y-values) to the original amount or the originating y-value. This is exactly the *percent of* increase that students first encounter in 7th grade (7.RP.3). The *3rd boss's* salary plan (from "The Summer Job") shows a growth rate from week 1 to 2 of $\frac{\$4-\$2}{\$2} = 1$, or 100% growth. The growth rate from week 4 to 5 is also 100% since $1 = \frac{\$32-\$16}{\$16}$. You might be more familiar with this term from the formula $A = P(1+r)^t$. The growth rate represents the "r" value, so since the growth rate of the 3rd boss's salary plan is 100%, *r*'s value would be 1.

However, the average rate of change in the salary amount with respect to the change in weeks from week 1 to 2 is $\frac{\$4-\$2}{(2-1)weeks} = \$2/week$, while the average rate of change from week 4 to 5 is $\frac{\$32-\$16}{(5-4)weeks} = \$16/week$. This distinction between growth rate and average rate of change can be supported by the percentage classification of the first and following the units in the second. Your science colleagues will be appreciative of encouraging students to keep units attached to quantities!

Attending to precision (SMP.6) with this vocabulary is key. Many students want to say exponential functions have a constant rate of change, but that is not true. Leading out of linear functions, it needs to be clear to students that *a constant rate of change is different from a constant rate of growth which implies multiplicative or proportional growth.*

QUESTIONS TO CONSIDER WITH COLLEAGUES

1. What are the connections between recursive relationships and "next – now" relationships?
2. How might you help your students understand the difference between *rate of change* and *rate of growth*?
3. What is the relationship between *rate of change* / *rate of growth* and *covariation* / *correspondence*? [See NC Math 1 Brief #2 for more information on covariation and correspondence views of functions.]

References:

Confrey, J., & Smith, E. (1994). Exponential functions, rates of change, and the multiplicative unit. Educational Studies in mathematics, 26(2-3), 135-164.

Oehrtman, M., Carlson, M., & Thompson, P. W. (2008). Foundational reasoning abilities that promote coherence in students' function understanding. Making the connection: Research and teaching in undergraduate mathematics education, 27-41.

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SUGGESTED CITATION

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