NC Math 1 – Quadratic Functions

NC MATH 1 –QUADRATIC FUNCTIONS

In NC Math 1, students begin to build an understanding of quadratic functions by examining key features and coefficients to factor quadratic expressions and solve quadratic equations in context. In NC Math 2, the standards extend this work to focus on: *creating* equations describing quadratic relationships; *analyzing* and *interpreting* equations by making connections across representations; *comparing* and *transforming* quadratic relationships, and introducing complex numbers.

The distinction between Quadratics within the two courses is not that students study Function in one course and add the Algebra in the second. Both sets of course standards call for connecting algebraic and graphical representations. Since the NC Math Standards first introduce quadratic functions in NC Math 1, students in this course are exposed to quadratics requiring less sophisticated algebraic maneuvering when compared to those in NC Math 2. For instance, on closer inspection of the standard A.REI-4, in NC Math 1, students rewrite "factorable" quadratic expressions and solve for real roots, considering them as zeros to the corresponding functions. In NC Math 2 students will engage with more sophisticated algebraic techniques including *completing the square* and using *the quadratic formula*, while continuing to be able to justify their algebraic reasoning in the solving method of their choosing.

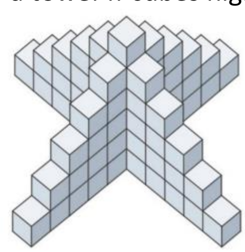
A VERTICAL VIEW OF QUADRATIC FUNCTIONS

When deciding how to begin this unit, it may be useful to start with a task that is accessible using approaches that build upon mathematics learning in 8th grade. Growing pattern tasks (such as those on visualpatterns.org) in which students must identify a pattern, generalize the pattern, and graph the relationship can be utilized to address numerous standards within this unit (e.g. NC.M1.A.SSE.1; F.BF.1b; A.CED.1; A.CED.2) and can help build upon students' growing understanding of rate of change, function families, and square numbers.

AN EXAMPLE: THE GROWING TOWER TASK

The figure below shows a tower built out of cubes. The height of the tower is measured by the center stack of cubes.

1. How many cubes are needed to build this tower?
2. How many cubes are needed to build a tower like this, but 12 cubes high? Justify your reasoning.
3. How would you calculate the number of cubes needed for a tower *n* cubes high?



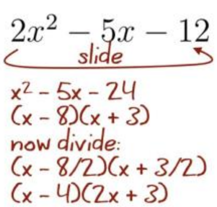
https://www.illustrativemathematics.org/content-standards/tasks/75

In NC Math 1, students engage first with linear and exponential functions, building an understanding of the distinction between their rates of change. That is, linear functions can be characterized by a constant rate of change, and exponential functions by a constant rate of growth that is proportional to the value of the function. While not explicit in the standards, many advocate that teachers can support students in extending their understanding of quadratics by designing instructional opportunities that highlight that quadratics have a linear rate of change (NCTM, 2014). For example, in the growing tower problem above, by examining different representations, students may recognize that the pattern has a rate of change (or first difference) that changes linearly and a second difference that is constant. Research has shown that students are indeed able to recognize the average rate of change over equal intervals of a quadratic function as linear (Lobato et al., 2012).

STANDARD AND FACTORED FORM

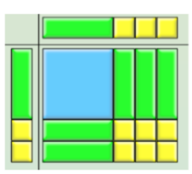
NC Math 1 students are required to only engage with the standard and factored forms of a quadratic expression. Algebra is a tool that students will use to factor quadratic expressions to determine the zeros of the corresponding quadratic function (A-SSE.3), the main purpose for factoring any polynomial. Students will need to be able to justify their process of solving quadratics using mathematical reasoning (A.REI.1). While many "tricks" have become popular for teaching factoring and may provide short-term results, it is important to develop students' conceptual understanding of factoring. There are some procedural methods (e.g., "Slide and Divide", "Rainbow", "Throwing the Football") that seem to make the process of factoring easier, when in reality they make the process more opaque and difficult for learners to understand the mathematics that is occurring. While mathematics teachers sometimes use these strategies to factor, they do not illuminate the patterns that a learner may use in choosing factors (e.g., signs of the factors, multiplication facts about the constant).

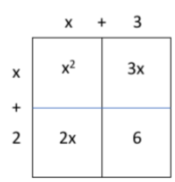
NIX THE TRICKS: *SLIDE AND DIVIDE* (Cardone, 2015)



In slide and divide, students factor a quadratic, but the middle steps are not equivalent. This gives students a false sense of what makes expressions equivalent and prevents them from seeing the structure in factored quadratic expressions (SMP 7)

Using geometric representations of multiplying and factoring binomials using algebra tiles (real or virtual) can help students make sense of the process of factoring in order to give them a foundation for factoring algebraic expressions that are more complicated. Another important connection to make with students learning to factor is the link between factoring by grouping and the distributive property. Using the "box method" (equivalent to factoring by grouping) to both multiply algebraic expressions and factor algebraic expressions and connecting it to the geometric representations afforded by algebra tiles is one way to draw attention to this connection between factoring and distributing. Students are familiar with the box method from their use of the array or area models used in learning multiplication and division in early grades.





$x^2 + 5x + 6$
$x^2 + 3x + 2x + 6$
$(x^2 + 3x) + (2x + 6)$
$x(x + 3) + 2(x + 3)$
$(x + 3)(x + 2)$

Equivalent Strategies for Factoring

UTILIZING TECHNOLOGY

Major work in NC Math 1 involves connecting various representations of mathematical relationships: verbal descriptions, tables of ordered pairs, algebraic expressions, and graphs (this can be seen in the repetition of standards across these units as the function types change). Technology allows students to quickly produce graphical representations, making the work of F-IF.7, 8, and 9 less daunting. Within these standards students will graph simple cases by hand but then use technology to analyze key graphical features and compare those features within different quadratics. Teacher Desmos offers several varieties of graphing tasks, including a quadratics bundle of 8 tasks that begins with an introduction to quadratics and ends with the usefulness of various quadratic forms.

Koklu and Topcu (2012) identified partial understandings students sometimes have about quadratic functions. For example, students may not realize that the coefficient c affects the graph of $ax^2 + bx + c$. Also, some students may think one of the parameters *a, b*, or *c*, is the "slope" of the parabola. They found that using dynamic programs (like GeoGebra or Desmos) can be a powerful way to create opportunities for students to visualize changes in quadratic functions since such software allows students to explore how the parameters affect the graph of the function.

QUESTIONS TO CONSIDER WITH COLLEAGUES

1. Factoring is known as the algebraic skill that just won't stick in high school. Why do you think that is? How might we avoid that consequence?
2. One key example of quadratics is projectile motion. What do a, b and c represent in that context?
3. Why might it be more beneficial to devote class time to guessing and checking appropriate factors instead of trying to memorize a non-sensical trick?

References

Koklu, O., & Topcu, A. (2012). Effect of Cabri-assisted instruction on secondary school students' misconceptions about graphs of quadratic functions. *International Journal of Mathematical Education in Science and Technology*, *43*(8), 999-1011.

Lobato, J., Hohensee, C., Rhodehamel, B., & Diamond, J. (2012). Using student reasoning to inform the development of conceptual learning goals: The case of quadratic functions. *Mathematical Thinking and Learning*, *14*(2), 85-119.

National Council of Teachers of Mathematics (NCTM). (2014). *Putting Essential Understanding of Functions into Practice 9-12*. Reston, VA: Author.

NC^{2}ML MATHEMATICS ONLINE

For more information on accessing Canvas learning modules or additional resources please visit http://nc2ml.org/

SUGGESTED CITATION

NC^{2}ML (2018, October). NCM1.4 Quadratic Functions. *Research-Practice Briefs*. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. Retrieved from nc2ml.org/brief-4