NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC^{2}ML Research-Practice Brief #5

NC Math 1 – Systems of Equations and Inequalities

A ***solution*** to many school mathematics problems is often a single value, a number. In this unit of the HS Instructional Framework students build on their understanding of a ***solution*** to a linear equation of two variables, as *an ordered pair that satisfies the equation* or *makes the equation true*.

An essential understanding of Unit 5 is that there exists an infinite number of ***solutions*** to linear equations and inequalities. Thus, students' work in linear equations and inequalities connects the *Algebra* and *Function* domains, by utilizing algebraic and graphical representations of ***solutions***.

To begin thinking about ***solutions to a system*** consider the solution to **5x + 3 = 28.** Graph together **y = 28** and **y = 5x + 3.** *How does the solution of that system relate to the solution of* ***5x + 3 = 28****? There may not be an infinite number of* ***solutions to a system*** *of linear eqautions, but could there be?*

GENERALIZING RELATIONSHIPS ACROSS 6-8

Since 6th grade, students have been generalizing mathematical relationships using variables. Initially, students utilize variables to create expressions from verbal descriptions or equations in which a variable represents a missing value. In middle grades, students also see variables in equations where they can truly vary (like in $\boldsymbol{a}+\boldsymbol{b} = \boldsymbol{b}+\boldsymbol{a}$) and variables describing relationships in which the value of one determines the value of another (in covarying relationships like $\mathbf{3}\boldsymbol{x}+\mathbf{4}\boldsymbol{y}=\mathbf{18}$). In addition, in 8th grade students build on prior knowledge to solve equations, inequalities, and a system of two linear equations.

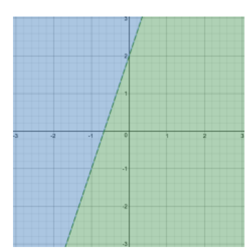
BUILDING FROM NC MATH 1 UNITS 1 & 2

This unit on linear systems is supported by and extends the content of functions and linear relationships in Units 1 and 2. Students will create graphical representations of linear equations and inequalities, while also connecting to the algebraic or symbolic representations. Tasks that ask students to create graphs from formulas, but also formulas from graphs can support students' understanding of how these distinct representations relate.

BUILDING A SYSTEM FROM CONTEXT

Students will be asked to create systems of linear equations and inequalities from contexts (NC.M1.A-CED.3) and will continue this creation of systems as they progress through NC Math 2 and NC Math 3. The distinction in NC Math 1 is that the focus is on linear relationships within a system. Specifically, students will take a given context, assign variables to represent quantities existing in that context, and then create mathematical equations or inequalities to represent how the context describes the relationships that exist between the variables. The system that is created models the context provided and allows students to graphically and symbolically investigate feasible variable values.

Extending the understanding of a solution to a single linear equation or inequality, students will investigate ***solutions to systems*** of linear equations or inequalities, or points that satisfy every relationship in a collection of more than one equation or inequality. It might be helpful to explicitly discuss that graphs of linear equations divide the plane into 3 distinct parts: the area where y > 3x + 2, the area where y < 3x +2, and the line where y = 3x+2.



EXAMINING THE STANDARDS AND INSTRUCTION

Despite the relatively short list of standards as compared with Units 1 and 2, the HS Instructional Framework recommends spending ample instructional time on this unit. Taking this recommendation provides opportunities for students to build flexibility in connecting and utilizing multiple representations in the analysis of a system, which requires considering multiple equations and/or inequalities and multiple graphs (NCTM, 2014). As the graph of a system may plainly show a solution to the system, "seeing" the solution within the formulas may be more challenging.

Students are expected to be able to algebraically manipulate variables within and across equations and inequalities, in order to find solutions. This skill may seem simple enough, since these systems will only involve linear relationships. Students who have been taught to manipulate variables with rote memorization of steps may struggle without opportunities to attach meaning to symbols and rate of change. See the NC Math 1 Briefs #2 & #3 for an explanation of covariation and rate of change.

Standard ***NC.M2.A-REI.5*** calls for students to: *Explain why replacing one equation in a system of linear equations by the sum of that equation and a multiple of the other produces a system with the same solutions.* For example, consider $x+y = 5$ and $x-y = 2$. The symbol "=" guarantees that each pair of expressions within the equations are the same, so that $(x+y)+(x-y)=5+2$. Which also means that x = 3.5. Further, since $x+y=5$, then $2(x+y)=10$. The essential understanding is that each of these manipulations build on the given equalities, so that the given equalities are still true and the relationships defined between the variables still hold. Students can then graph these new relationships to visually compare how the new equations relate to the original system.

TIME FOR TECHNOLOGY

An affordance of NC Math 1 Units are the multiple opportunities teachers have to support students in gaining experience in both graphing functions by hand and by using technology. The content of this unit is well suited to the use of a graphing utility and most utilities have the capability to graph inequalities as well. Two examples are provided below.

1) *TI graphing calculator*: press the **Y=** button to enter the function to be graphed, then left-arrow over to the backslash symbol to the left of "Y1" and press ENTER to scroll through the options for choosing the shading appropriate for the inequality desired.
2) *Desmos* (desmos.com): choose the appropriate inequality on the keypad or use the keyboard as you create the function to indicate the inequality.

Research has shown that students may have difficulties connecting graphs with the physcical context they are representing and that students need ample opportunities to develop proficiency in interpreting graphs (McDermott et al., 1987). Regardless of the technology utilized, providing students opportunities to use technology to graph relationships with more efficiency allows more time for analyzing systems in context and continue building conceptual understanding of systems, rate of change, and covariation.

EXAMPLES: USING DESMOS ACTIVITIES TO DEEPEN UNDERSTANDING OF LINEAR SYSTEMS

Desmos has a nice bundle of activities for systems of linear equations, each designed to deepen student understanding of important concepts. For example, in the Systems of Two Linear Equations task, students explore the numerical and graphical meaning of "solution." In the Wafers and Crème task, students build a system from a context and work to find the solution. Students have the opportunity to really stop and think about the relationships prior to assigning variables and working toward a precise solution.

QUESTIONS TO CONSIDER WITH COLLEAGUES

1. Traditionally, there are three methods to solving a system of equations (i.e., elimination, substitution, or graphing). How does manipulating equations like (x + y) + (x – y) = 5 + 2 fit in with these methods?
2. How do we give students freedom in choosing solving methods to provide efficiency while also providing an opportunity for learning?
3. What do you anticipate would happen if you stopped and graphed every new relationship you created in each of the steps as you were finding a solution to a system? (e.g., Try graphing the example in the paragraph to the left.)

References

McDermott, L., Rosenquist, M.L., & van Zee, E.H. (1987). Students difficulties in connecting graphs and physics: examples from kinematics. *American Journal of Physics*, *55*(6).

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all*. Reston, VA: National Council of Teachers of Mathematics.

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NC²ML MATHEMATICS ONLINE

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SUGGESTED CITATION

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