NC Math 2 - Transformations

GEOMETRIC TRANSFORMATIONS

Transformations are the focus of the first unit of NC Math 2, within the HS Instructional Framework. Traditionally, transformations were seen as actions applied to geometric objects without connections to functions, beyond the fact that their pre-images and images may be drawn within the coordinate plane. It also may have then been noted that rigid transformations produce images that are congruent to their pre-image and dilations create images that are similar, but not congruent (unless you applied an uninteresting scale factor of 1).

Transformation content appears in both the NC Middle School Mathematics Standards and NC High School Mathematics Standards. Students begin learning about location and representations within the coordinate plane in 5th grade and are introduced to translations, rotations, and reflections in 8$^{th.}$ The HS Instructional Framework highlights connections between 8th grade, NC Math 2, and the pre-requisite experiences for understanding transformations as functions that can verify congruency or similarity.

The NC Math 2 Transformations Unit connects the *Function* and *Geometry* domains and is described by NC.M2.F-IF.1 and NC.M2.F-IF.2 standards.
Recall the definition of a function:

> *A relation that pairs every element in one set, called the domain, with exactly one element of a second set, called the range.*

These standards require students to expand the idea of the types of elements that could be inputs by allowing the domain of a function to be a collection of ordered pairs. That is, instead of an input element that looks like ***x***, a transformation would have an input element that looks like **(*x,y*)**. Similarly, the range of a transformation is also a set of ordered pairs.

NC.M2.F-IF.2, is very explicit in extending the function notation (think **f(*x,y*)** instead of **f(*x*)**) to very specific transformation examples. This list of examples excludes many of the transformations that students can draw, because those transformations may be challenging to formalize with algebraic formulas.

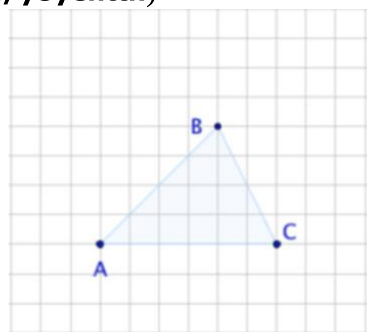
For example, the standard requires the use of function notation to express "rotation by multiples of 90 degrees about the origin". In general, rotations about the origin can be shown as the results of a transformation defined by multiplying a position by a 2x2 matrix. Also, it is possible to write function formulas that translate points through rotations of other degrees, but those generalizations are better done with polar coordinates. Both polar coordinates and matrix multiplication are beyond the scope of NC Math 2, thus this standard is very specific.

Mapping Notation Approach Examples	
$r_{x\text{-}axis}(x\,,\,y) = (x\,,\,-y)$	Reflection (in x – axis)
$T_{a,b}(x\,,\,y) = (x+a\,,\,y+b)$	Translation
$R_{0,90^\circ}(x\,,\,y) = (-y\,,\,x)$	Counter-clockwise Rotation 90° about origin
$D_{0,\,k}(x\,,\,y) = (kx\,,\,ky)$	Dilation scale factor k

Along with the two Function standards in this unit, the remaining standards fall into the Geometry domain. NC.M2.G-CO.2-.6 call for students to experiment with rigid and non-rigid transformations and to verify relationships between pre-images and images through experimentation. This use of experimentation is consistent through the remaining standards.

AN EXAMPLE: DILATIONS TASK

One way to introduce this unit is to focus on the effects of dilation and their respective centers on a figure in the coordinate plane. (**tinyurl.com/y8yehtth**)



a. Draw a dilation of ABC with:
   i. Center A and scale factor 2.
   ii. Center B and scale factor 3.
   iii. Center C and scale factor 12.
b. For each dilation, answer the following questions:
   i. By what factor do the base and height of the triangle change? Explain.
   ii. By what factor does the area of the triangle change? Explain.
   iii. How do the angles of the scaled triangle compare to the original? Explain.

LOOKING AHEAD: NC MATH 2 – UNIT 4

The standards call for a shift from the experimentation of Unit 1, into proving geometric theorems in Unit 4. The language of the standards and the structuring and ordering of Units 1 and 4 allow students time to explore, so that they can begin to conjecture about variance and invariance around geometric objects.

Looking at upcoming units, there exists a lot of overlap between Units 1 and 4, particularly with NC.M2.G-CO.6 and NC.M2.G-SRT.1. In Unit 4, students gain a more formal understanding of transformations and their importance for proving similarity and congruence.

"Writing a formal proof of a geometric result is the endpoint of a significant piece of mathematical investigation. It is not generally an activity to be undertaken on its own. In particular, it occurs *after* an invariance has been detected, conjectured, and tested against a context of variation; *after* an appropriate diagram has been constructed and understood; and *after* relevant definitions have been brought into play." *(*Sinclair, Pimm, & Skelin, 2012*)*

Given the importance of students' exploration and experimentation, this unit provides the first seeds for justification and discovery of which attributes vary and which ones are invariant that will later be key in proving geometric relationships. Traditionally, students' mathematical tools for exploring whether attributes vary through construction consisted of a straightedge and compass. Students may also use mirrors for reflection, tracing paper for moving or folding, and geometry apps and software such as geogebra.com or desmos.com/geometry. In the Desmos Transformations Golf task, students are asked to experiment to transform figures within the coordinate plane. (**tinyurl.com/yd7takqu)**

These dynamic technology tools also support students' transition to include ordered pairs as inputs in the domain (Hollebrands, 2003). These tools allow students to drag a construction so that they see that the input could be any point in the plane, supporting student understanding of transformations not as motions, but as functions.

QUESTIONS TO CONSIDER WITH COLLEAGUES

- *What conceptions do your students commonly have about tranformations as functions?*
- *How could an attention to domain support them?*
- *How can you design appropriate interventions or tasks to advance or refine their conceptions?*
- *What questions should be asked in the Dilations Task so that the task better addresses NC.M2.G-SRT.1?*

References

Hollebrands, K. F. (2003). High school students' understandings of geometric transformations in the context of a technological environment. *The Journal of Mathematical Behavior*, *22*(1), 55-72.

Sinclair, N., Pimm, D., & Skelin, M. (2012). *Developing essential understanding of geometry for teaching mathematics in grades 9-12*. National Council of Teachers of Mathematics.

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SUGGESTED CITATION

NC²ML (2018, October). NCM2.1 Transformations. *Research-Practice Briefs*. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. Retrieved from nc2ml.org/brief-7