
# NC Math 2 - Quadratics

## DISTINGUISHING QUADRATICS IN NC MATH 1 & 2

In NC Math 2, the [HS Instructional Framework] Unit 2 standards extend the work of quadratics in NC Math 1 by focusing on: *creating* equations describing quadratic relationships; *analyzing* and *interpreting* equations by making connections across representations; *comparing* and *transforming* quadratic relationships, and an introduction to complex numbers.

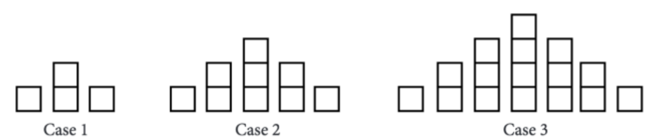
In NC Math 1, students engage first with linear and exponential functions, building an understanding of their respective rate of change and rate of growth. That is, linear functions can be characterized by a constant rate of change, and exponential functions by a rate of change that is proportional to the value of the function. While not explicit in the standards, many advocate that teachers can support students in extending their understanding of quadratics by designing instructional opportunities that highlight that quadratics have a linear rate of change (NCTM, 2014). For example, in many growing pattern tasks, students examine visual representations and may recognize that the pattern has a rate of change (or first difference) that changes linearly and a second difference that is constant (see the Squares Upon Squares example).

The distinction between Quadratics within the two courses is not in their blend of *Algebra* and *Function*. Both sets of course standards call for connecting algebraic and graphical representations. Instead, the algebraic maneuvering becomes more sophisticated in NC Math 2. In NC Math 1, students rewrite "factorable" quadratic expressions and solve for real roots, considering them as zeros to the corresponding functions. In NC Math 2 students engage with more sophisticated algebraic techniques including *completing the square* and *the quadratic formula*, while continuing to be able to justify their work.

## AN EXAMPLE: THE SQUARES UPON SQUARES TASK

When deciding how to begin this unit, it may be useful to start with a task that is accessible using approaches that build upon mathematics learning from the previous course. Growing pattern tasks allow students to visualize different representations of shapes and how they relate to the algebraic and graphical forms. In these types of problems, students must look for, describe, and generalize a pattern. Such tasks can be utilized to address numerous standards within this unit (e.g. NC.M2.A.SSE.1; F.BF.1; A.CED.1; A.CED.2) and can help build upon students' growing understanding of rate of change and function families.

**How do you see the shapes growing?**




https://www.youcubed.org/task/squares-upon-squares/

In addition to describing this visual pattern, students may recognize that the number of squares within each shape grows at a linear rate in the 1st difference, with a constant 2nd difference which is shown in the table below.

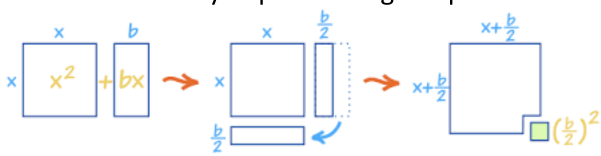
| Case # | # of squares | 1st difference | 2nd difference |
|---|---|---|---|
| 1 | 4 | | |
| 2 | 9 | 5 | 2 |
| 3 | 16 | 7 | 2 |
| 4 | 25 | 9 | |

## STUDENTS' THINKING ABOUT QUADRATICS

Extending from the growing shapes example, research has shown that students are able to recognize the average rate of change over equal intervals of a quadratic function as linear (Lobato et al., 2012). Similarly, they are also able to recognize symmetry; however, they may struggle in making sense of symmetry conceptually and in understanding the relationship between quadratic expressions, equations, and functions (Nielson, 2015). Supporting their understanding of symmetry, students should be given opportunities to make connections between graphical and algebraic representations of quadratic relationships. An excellent lesson for connecting ways that different forms of a quadratic can be used to highlight key characteristics of its graph can be found at the MARS Project, Representing Quadratic Functions Graphically.

## COMPLETING THE SQUARE

Another point of instructional decision-making will arise when introducing the algebraic technique of *completing the square*. Well-designed curricular materials highlighting a visual approach can be used to show students that this process can actually **complete** a **square**, while maintaining the original equality. This understanding can be cultivated by using algebra tiles (either real or virtual). A visual representation of this algebraic process can bring meaning to the symbolic manipulation and to why new expressions are equivalent to the original. Such experiences can support students in successfully implementing the procedure.

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## QUADRATIC FORMULA

When introducing the *quadratic formula*, drawing from standard ***NC.M2.A-REI.4***, teachers could decide to derive the formula from a quadratic polynomial written in standard form. Beginning with

$$ax^2 + bx + c = 0$$

subtracting ***c*** from both sides of the equation and dividing the equation by the factor ***a*** results in

$$x^2 + \frac{xb}{a} = -\frac{c}{a}$$

Applying the procedure of completing the square to this equation, attending to maintaining equality, and then solving for **x** results in the quadratic formula.

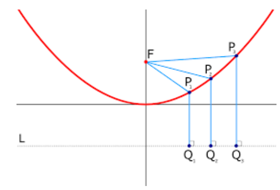
The goal of sharing such a derivation is not to have students memorize steps, but rather to connect what students know to this new relationship. Making sense of ***why*** the quadratic formula works helps to ensure students understand when to use it and how to use it effectively. Since every NC Math 2 class of students is unique, it is up to teachers to decide how to best utilize this derivation (investigation, whole group line-by-line discussion, jigsaw, etc.), in order to maximize students' understanding.

## SOLUTIONS – ROOTS, ZEROS, & COMPLEX NUMBERS

Often times, we use the terms roots and zeros synonymously; however, they are not exactly the same thing. A *root* of a polynomial is a value that makes the polynomial expression equal to 0. A *zero* of a polynomial function is the input that outputs a function value of 0, thus a zero of a function is the x-coordinate of an x-intercept on the graph of the function. As we use these terms with our students, it is important that we *attend to precision* (*SMP 6*) so that students are clear of the distinction.

Every quadratic expression has at most two roots. But it's not the case that every quadratic function has zeros. As students investigate a variety of quadratic functions, they will be introduced to a new set of numbers, the set of complex numbers. Built from the set of real numbers using the imaginary unit ***i***, complex numbers provide solutions to equations of the form $ax^2+bx+c=0$ that do not have real number solutions. In NC Math 2, students will be calculating solutions, connecting the presence of real or complex solutions to the graphical representation of the quadratic function.

## EXTENSION – MATH KNOWLEDGE FOR TEACHING



What is your definition of a ***parabola***? Many would say that a ***parabola*** is the name of the shape given to the graph of a quadratic function. A more formal definition would be that a ***parabola*** is the collection of all points in the plane equidistant from a given point *F*, the *focus*, and a given line *L*, the directrix (in other words, $\overline{FP_1} = \overline{P_1Q_1}, \ldots$). The shape is one of four called *conic sections*, since it can be created by intersecting a cone with a plane. Students will make explicit connections between quadratics and conic sections in precalculus.

## QUESTIONS TO CONSIDER WITH COLLEAGUES

1. How does the use of algebra tiles build students' conceptual understanding while strengthening their procedural understanding?
2. How do we assist students with using appropriate tools strategically when choosing a solution pathway for solving a quadratic without limiting their autonomy?


## References

Lobato, J., Hohensee, C., Rhodehamel, B., & Diamond, J. (2012). Using student reasoning to inform the development of conceptual learning goals: The case of quadratic functions. *Mathematical Thinking and Learning*, *14*(2), 85-119.

National Council of Teachers of Mathematics (NCTM). (2014). *Putting Essential Understanding of Functions into Practice 9-12*. Reston, VA: Author.

Nielsen, L. (2015). *Understanding quadratic functions and solving quadratic equations: an analysis of student thinking and reasoning* (Unpublished doctoral dissertation). University of Washington, Seattle, WA.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## NC²ML MATHEMATICS ONLINE

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## SUGGESTED CITATION

NC²ML (2018, October). NCM2.3 Quadratics. *Research-Practice Briefs*. North Carolina Collaborative for Mathematics Learning. Greensboro, NC. Retrieved from nc2ml.org/brief-9