NC COLLABORATIVE FOR MATHEMATICS LEARNING

NC²ML Research-Practice Brief

Cross Multiplication: To Teach or Not?

WHAT IS WRONG WITH BEING PROFICIENT WITH RULES AND TECHNIQUES?

Many teachers in today's classrooms were taught mathematics as a set of procedures, or "tricks", to solve mathematical problems quickly and efficiently. If it worked for today's teachers, what is wrong with teaching them to current students? Using clever mnemonics such as *Please Excuse My Dear Aunt Sally* to remember the order of operations and *Flip the guy and multiply!* or *Keep, Change, Flip* to divide fractions can be helpful for solving problems. While these strategies help us remember procedural rules for solving, memorizing tricks without meaning is not "doing mathematics". In *Principles to Action*s (2014), the National Council of Teachers of Mathematics (NCTM) noted that the learning of procedures without the conceptual understanding of mathematics is one reason why mathematics education in the U.S. has not advanced in recent years. Additionally, researchers argue that applying cross multiplication without meaning to solve proportions does not equate to understanding the underlying ideas of proportional thinking (Hoffer, 1988).

HOW DOES CROSS MULTIPLICATION WORK?

Figure 1. Challenge: Explain why cross multiplication works.

$$\frac{a}{b} = \frac{c}{d}$$

Cross multiplication is a mechanical procedure that students have been taught in the past to identify equivalent fractions or find the unknown in a proportional relationship between two quantities. To identify proportionality, students multiply the "top number" of each ratio by the "bottom number" of the other ratio and compare the two products (see Figure 1). For example, when comparing the ratios, 2:3 and 4:6, students would multiply 2 and 6, and 3 and 4. Since the products are equal (12), students can safely assume the ratios are proportional to one another. Issues arise, however, when students try to make meaning of ratios that are not proportional. For example, if the cross multiplication results in two products that are unequal, what does that mean about their proportionality? Which ratio is greater, and by how much? While the procedural method of cross multiplication reveals equivalency, or proportionality, it does not address (1) how the ratios are proportional or (2) how to make further meaning of disproportionality. Additionally, Battista & Van Auken Borrow (1995) argue that students are not ready to understand why cross multiplication works until they have studied properties of algebra (see Figure 2) which tends to be later than middle school. Therefore, middle school students should develop other, personally-meaningful ways to solve simple proportion problems.

Figure 2. Why cross multiplication works.

$$(d)\,\frac{a}{b} = \frac{c}{d}\,(d)$$

1. **Multiply both sides by d. By the multiplicative inverse property (d)($\frac{1}{d}$) =1, ad/b = c.**
2. **Now, multiply both sides of ad/b = c by b. By the multiplicative inverse property (b)($\frac{1}{b}$) =1, ad = bc.**

UNDERSTANDING RATIOS AND PROPORTIONS

Understanding situations involving ratios and proportions conceptually requires a strong sense of numbers and operations as well as an understanding of how quantities are related. More specifically, students need to know about the multiplicative relationships that exist between two proportional quantities. To build this conceptual understanding, proportions are commonly taught in contexts. Consider the word problem in Figure 3.

The student who only knows how to find the answer by setting up a proportion to cross multiply will be left with the task of setting up a structure such as a/b = c/*d,* and then correctly "plugging in" numbers. In this case, students would want to know if 2/3 = 8/9. They would find that 18 is not equal to 24, but not know where to go from there. Simple "plugging in" numbers from a context into the a/b = c/*d* structure will often result in mistakes (e.g., inserting the incorrect numbers) or uninterpretable calculations if the student does not first consider how the quantities relate.

Figure 3. A typical word problem

There are two recipes for orange punch given below. Which mixture gives the least "orangey" flavor?

Mix A	*Mix B*
2 cups orange juice	*8 cups orange juice*
3 cups lime soda	*9 cups lime soda*

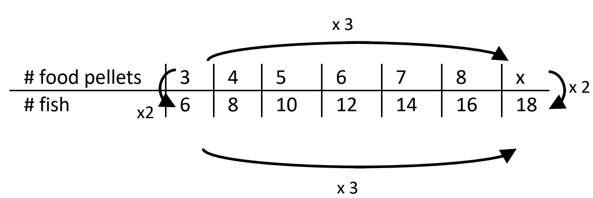
TEACHING PROPORTIONS

Battista and Van Auken Borrow (1995) argue that, before comparing ratios, a ratio itself should represent a multiplicative link between two quantities. In other words, 2 cups OJ for every 3 cups lime soda should mean that the unit of 2 cups OJ is always linked with 3 cups lime such that if the OJ is doubled, so is the lime soda. Additionally, the amount of lime juice is always 1.5 times greater the amount of OJ for this recipe and the amount of OJ is always 2/3 the amount of lime soda. These multiplicative relationships between the two quantities in ratios are critical for a rich understanding of ratios and proportions. Battista Van Auken Borrow (1995) also offer a possible learning route for students: 1. Construct a ratio as multiplicatively linked quantities, 2. Build up strategies (2/3 is equivalent in taste to a 4/6 recipe, and a 6/9, 8/12, etc. Students build up by adding 2 cups OJ and 3 cups lime, whilst preserving the link), 3. Abbreviated build up (2/3 is the same taste as a 16/24 because you multiply both liquid quantities by 8), and 4. Conventional proportion notation (2/3 = 16/24).

NOTATIONAL SUPPORT

Too frequently, teachers and textbooks emphasize procedural skills with formal algorithms instead of building students' proportional reasoning (Cramer, Post, & Currier, 1993). Introducing proportions with formal algorithms results in missed opportunities to scaffold students' thinking about proportions with less formal representations. According to Webb, Boswinkel, and Dekker (2008), one way teachers can illustrate relationships between quantities is by using informal (e.g., illustrations, visual models) and pre-formal representations, such as ratio tables or double number lines, instead of jumping to formal structures such as a/b = c/d. They argue that students who have not been exposed to informal or pre-formal representations will unlikely be able to use proportional reasoning to solve a formal proportion using cross-multiplication. Additionally, informal models and pre-formal representations create more opportunities for students to construct proportional relationships, thus promoting increased number sense and proportional reasoning.

Teachers can capitalize on students' building up strategies by introducing a ratio table or double number line to organize their work. Then, students will naturally see relationships within the "long ratio table" such as a) as the # of pellets grow by 1, the # of fish grow by 2 b) There are always twice as many fish as food pellets, and c) to find x, triple 6 fish (to get 18 fish) and triple 3 pellets to find 9 pellets. Arrow notations showing scale factors horizontally and the constant of proportionality vertically can be effective.



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