Precalculus Collaborative Instructional Framework

The following Collaborative Instructional Framework is meant to serve as a guide for teachers and districts as they organize the curriculum for the school year. Unlike traditional pacing guides, the instructional framework consists of clusters of standards that are meant to be adapted to various schools and contexts. The instructional framework used research on students' learning progression in mathematics to create and order clusters of standards that are taught together. While there is a strongly suggested order for teaching the clusters, we recognize that schools differ in their contexts and may wish to switch the order around. In those cases, we have given guidance regarding alternative clusterings; however, we note when certain clusters need to be taught in a certain order.

The Collaborative Instructional Framework was created over an eight-month period, beginning in July 2019. Twenty-eight classroom teachers, district leaders, and university faculty worked together to a) discuss research and practice related to pacing guides, student learning progressions, and standards, b) determine the best clusterings of the standards based upon research when possible, and c) draft the framework. The team solicited feedback on this initial draft, first at the NCCTM conference and through an online survey in November 2019. The team reviewed all feedback received to consider which suggestions could be addressed by the framework. A second draft of the instructional framework was made available in late January through February 2020 for additional feedback, which is incorporated in this final version.

The objectives for Precalculus have remained unchanged for almost 20 years. The team recognizes that the depth and breadth of the course may feel like a significant shift from what was practiced in the past. For example, many educators offering feedback on the drafts noted that polar coordinates, vectors, and conic sections were underdeveloped. Like the NC Mathematics Standards, the Instructional Framework addresses the minimal expectations of students preparing for the first level of Calculus. If time allows, teachers should enhance and extend the content of the course as needed to meet the needs of students.

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Standards for Mathematical Practice are critical ways of acting and communicating in classrooms that should be instilled in students throughout the school year. Whether students are learning to reason proportionally or statistically, they should be obliged to make sense of the problems posed (SMP1) and create a mathematical solution that can contribute to their peers' and their own learning (SMP3). When solving a problem, such as which company is the cheapest when comparing the prices of t- shirts, students should be able to create a viable argument for their choice, with mathematical evidence to defend their solution (SMP3). Students should be able to move among various representations, reasoning quantitatively with symbols (SMP2), and create models of both every day and mathematical situations they encounter (SMP4). Teachers should provide opportunities for students to reason with a variety of tools (SMP5), including technologies that are specific to mathematics (e.g., calculators, Desmos, GeoGebra, etc.). Attending to precision (SMP6) is a practice in which students attempt to present clear arguments, definitions, and meanings for symbols as they explain their reasoning to others. Finding patterns and structure is crucial throughout the standards as students attempt to mathematize complex problem situations (SMP7). Finally, students should attempt to find regularity in repeated reasoning (SMP8), as with this repetition they are able to generalize their findings from one instance to multiple instances.

Precalculus Cluster Sequencing

The clusters are recommended using the progression in the framework, but this is not the only possible progression teachers may use. Please look in the "What is the mathematics?" and the "Important Considerations" for notes about the purposeful sequencing of the clusters, if another cluster is desired. Also, continue to focus on how the Standards of Mathematical Practice can be incorporated with these content clusters. If your school or district is considering changing the order of the clusters, we suggest keeping most of the sequence with the following two alternative orderings:

Recommended Order	Alternative Order A	Alternative Order B
Building Mathematical Community with Parent Functions & Key Features Extending Polynomial & Rational Functions Composition & Inverse Functions Exponential & Logarithmic Functions Trigonometry Parametric Equations ACT Prep*	Building Mathematical Community with Parent Functions & Key Features Extending Polynomial & Rational Functions *Exponential & Logarithmic Functions* *Composition & Inverse Functions* Trigonometry Parametric Equations ACT Prep*	Building Mathematical Community with *ACT Prep Content* Extending Polynomial & Rational Functions Composition & Inverse Functions Exponential & Logarithmic Functions Trigonometry Parametric Equations

The recommended order of the clusters is based on research about the mathematical progressions of learning. In the Building Mathematical Community Unit, students use the content of Parent Functions & their key features to develop class norms and a positive culture that is conducive of the Standards of Mathematical Practice. This serves to assist students in recalling prior knowledge about function families before moving into the next cluster, Extending Polynomial and Rational Functions. The next cluster of Composition & Inverse Functions allows students to look for and make use of structure (SMP7) as they develop procedural skills surrounding functions. The Exponential & Logarithmic Functions cluster develops students' understanding of Composition & Inverse Functions as they look for regularity in repeated reasoning (SMP8). Students then follow with a study of the Trigonometry & Parametric Equations clusters, as they make sense of new problems and persevere in solving them (SMP1) while modeling with mathematics (SMP4). The last cluster of ACT Prep serves as a floating cluster and is accompanied with the following note: *This unit does not have to be taught at the end of the* *course. These standards can be scattered anywhere it works for you to teach them. They do not fit necessarily in any of the other units cohesively, but that does not mean you couldn't throw them in where you have time or want to.* This floating unit serves to remind educators of their autonomy in placing the ACT Prep standards where they best fit for their context. It is the suggestion of the authors of this document that the content in the ACT Prep Unit be spread across the course. For instance, it may make sense to include a day or two on operations of complex numbers in the Extending Polynomial & Rational Functions cluster. It may also make sense to include a day or two on vectors in the Parametric Equations cluster (e.g. s*ee One Shot Task*).

In Alternative Order A, the Exponential & Logarithmic Functions cluster precedes the Composition & Inverse Function cluster, as schools and districts may choose to have students experience an example of two inverse functions in context, developing the conceptual knowledge of Composition & Inverse through observation. This would then be followed by the procedural knowledge development in the Composition & Inverse cluster to expand the conceptual knowledge to other function families.

In Alternative Order B, the ACT Prep content (complex numbers, matrices, and vectors) serves as the medium for the Building Mathematical Community Unit. This order allows students to be exposed to the mathematics of the ACT prep unit at the beginning of the course for those educators who teach a population for whom this would be helpful.

Course Pacing Overview:

Unit Name	Duration
Building Classroom Community with Parent Functions & Key Features	2 - 3 days
Extending Polynomial and Rational Functions	4 weeks
Composition and Inverse Functions	1.5 - 2 weeks
Exponential and Logarithmic Functions	3.5 - 4 Weeks
Trigonometry	4 - 4.5 weeks
Parametric Equations	3 days
ACT Prep Unit	2 days

Unit Name: Building Classroom Community

Duration: Approximately 3 days

Standards (Content):
It is recommended that the first week of the school year be spent engaging students with open-ended mathematics problems designed to support the students' growth mindset. This first week is also an opportune time for setting up the classroom expectations and norms for collaborating with classmates and participating in whole class discussions. Using this week strategically, this cluster is designed to also review prior knowledge involving Parent Functions & Key Features of functions. This grounds the work of building mathematical community in a content area that will serve both the needs of the students (to review) and the teacher (in terms of pacing).

Mathematical Practices:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

What is the Mathematics?
- Recognize constant, linear, exponential, quadratic, square root, cubic, and absolute value functions from multiple representations including tables, graphs, function rules, and verbal descriptions.
- Discuss the key features (including domain, range, intervals where the function is increasing, decreasing, positive, or negative, end behavior, minimum and maximum) from Math 1-3 courses.
- Develop mathematicians with positive attitudes about their ability to do mathematics by:
    - Creating opportunities to develop an appreciation for mistakes
    - Seeing mistakes as opportunities to learn
    - Teaching students to take responsibility for their learning
- Develop mathematicians who respect others by:
    - Demonstrating acceptance, appreciation, and curiosity for different ideas and approaches
    - Establishing procedures and norms for productive mathematical discourse
    - Consider other solution paths
- Develop mathematicians with a mindset for problem solving by:
    - Encouraging student authority and autonomy when problem solving

- Emphasizing questioning, understanding, and reasoning about math, not just doing math for the correct answer
- Asking follow-up questions when students are both right and wrong
- Allowing students to engage in productive struggle and moving them along by questioning, not telling

Important Considerations:
For success, significant time should be spent setting up the classroom culture. This includes:

- Developing classroom norms for communication (ex: non-verbal signals, listening and speaking expectations, talk moves for math discussions)
- Developing math routines
- Setting various expectations for the structure of the math block (ex: expectations for whole class instruction, cooperative learning, independent learning, effective integration of technology, etc.)
- Math discourse needs explicit modeling and practice. This includes students:
    - Sharing their thinking
    - Actively listening to the ideas of others
    - Connecting to others' ideas
    - Asking questions to clarify understanding
- Mathematical norms: http://www.youcubed.org/wp-content/uploads/Positive-Classroom-Norms2.pdf

Formative Assessments/Tasks:

- Parent Functions Card Sort http://www.mrseteachesmath.com/2014/12/parent-functions-matching-activity.html
- Parent Function Polygraph: https://teacher.desmos.com/polygraph/custom/560ad6907701c30306330608
- https://www.insidemathematics.org/sites/default/files/materials/sorting%20functions.pdf
- Exponential Marble Slides: https://teacher.desmos.com/activitybuilder/custom/566b317b4e38e1e21a10aafb
- Linear Marble Slides: https://teacher.desmos.com/activitybuilder/custom/566b31734e38e1e21a10aac8
- Quadratic Marble Slides: https://teacher.desmos.com/activitybuilder/custom/566b31784e38e1e21a10aade
- Which One Doesn't Belong: http://wodb.ca/graphs.html Graphs 16, 22, 27
- Key Features of a Function exploration: https://teacher.desmos.com/activitybuilder/custom/564b8b6fa8e7fefa0bad36b7
- See https://www.youcubed.org/ for suggested activities on building classroom community

Unit Name: Extending Polynomial and Rational Functions
Duration: Block: Four Weeks 22-24 days including assessment
Standards (Content): F.4.6: Implement graphical and algebraic methods to solve optimization problems given rational and polynomial functions in context with support from technology. A.1.2 Implement graphical methods to solve rational and polynomial inequalities. F.4.5: Interpret algebraic and graphical representations to determine key features of rational functions. *Key features include: domain, range, intercepts, intervals where the function is increasing, decreasing, positive or negative, concavity, end behavior, continuity, limits, and asymptotes.* A.1.1: Implement algebraic (sign analysis) methods to solve rational and polynomial inequalities. F.4.8. Identify the conic section (ellipse, hyperbola) from its algebraic representation in standard form. F.4.9. Interpret algebraic and graphical representations to determine key features of conic sections (ellipse: center, length of the major and minor axes; hyperbola: vertices, transverse axis; parabola: vertex, axis of symmetry).
Mathematical Practices: 1. **Make sense of problems and persevere in solving them** 2**. Reason abstractly and quantitatively** 3. Construct viable arguments and critique the reasoning of others 4. **Model with mathematics** 5**. Use appropriate tools strategically** 6. Attend to precision 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

What is the Mathematics?

- Emphasis in this unit should be on the idea that many concepts they initially learned in M1-M3 we will now extend and apply to varied situations. (i.e. discussing domain in context for polynomials and rational functions).
- Throughout the unit, the key features listed in 4.5 should be addressed, but every key feature does not need to be included in every problem.
- The standards related to Conic Sections include parabolas, ellipses, and hyperbolas in standard form. In Math 3, students "Translate between the geometric description and the equation for a conic section. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation" (M3.G-GPE.1). Here, we build on these ideas so that Students can identify each based on its algebraic representation in standard form, identify the key features included in F.4.9, and use these to graph conic sections both by hand and with technology.
- Solving inequalities can be motivated by finding domains of square roots of rational/polynomial functions that leads to the sign analysis (A.1.1).
- For the benefit of student learning and engagement, utilize multiple representations (tabular, algebraic, graphical) to complement each standard within this unit.

Important Considerations:

- Launch the unit with an optimization problem as an interesting hook (F.4.6). This can lead into theoretical work specific to finding domains of functions and solving inequalities, then come back to this standard throughout the unit (classic box problem, cone, zip line task).
- Students are asked to *both* build models given optimization problems *and* to analyze models given to them in context. Students can use technology to find extreme values of the function.
- Development of "sign chart" approach to characterizing positive/negative intervals (for use with inequalities, A.1.1), make connections to graphical representations of polynomial/rational functions (A.1.2).
- In Math 3, students work with rational expressions and solving rational equations when the denominator is linear (see for example, M3.A-APR.7, M3.A-REI.2, M3.F-IF.7, and M3.F-IF.9). Now, the denominator may not be linear, so students should explore how this change affects key features including domain, range, intercepts, intervals, concavity, end behavior, continuity, limits, and asymptotes.
- In this early unit, limits should be addressed in terms of end behavior of functions or

the behavior of functions near vertical asymptotes. The emphasis should be on the qualitative behavior of functions. Notation will be introduced, and concepts formalized in later units. ● Students should use both algebra and technology to understand continuity informally.
Global Perspectives ● Using optimization problems here sets students up for further exploration of optimization contexts in Calculus. ● From an algebraic standpoint, we start with optimization so we can talk about domain, etc, and inequalities can be analyzed using sign analysis. While these same ideas can easily be determined using technology, the algebraic procedures are critical for derivative tests in Calculus. ● A significant amount of time in this unit should be devoted to key features of rational functions (4.5) as this builds a foundation for exploration of the behavior of functions in Calculus around first and second derivatives.
Formal Assessments/Tasks: **Zipline Task (optimization problem)** **Key features of graphs: Lake Sonoma Task (Illustrative Mathematics)** **Polygraph: Conics (Desmos)** **Building Conic Sections (Desmos)** **Polygraph: Rational Functions (Desmos)** **Solving Polynomial Inequalities (Desmos)**

Unit Name: Composition and Inverse
Duration: **Block:** 1.5 Weeks 8-10 days including assessment
Standards (Content): F.5.1: Implement algebraic procedures to compose functions. F.5.2: Execute a procedure to determine the value of a composite function at a given value using algebraic, graphical, and tabular representations. F.5.3: Implement algebraic methods to find the domain of a composite function. F.5.4: Organize information to build models involving function composition. F.5.5: Deconstruct a composite function into two functions. F.5.6: Implement algebraic and graphical methods to find an inverse function of an existing function, restricting domains if necessary F.5.7: Use composition to determine if one function is the inverse of another function.
Mathematical Practices: **1. Make sense of problems and persevere in solving them** **2. Reason abstractly and quantitatively** 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Use appropriate tools strategically 6. Attend to precision **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**
What is the Mathematics? ● Students will use the knowledge of evaluating functions at a specific value to extend into composition of functions. They can build on the discussion of function families to compose and get new functions. ● Understanding that one can take the "output" of one function and use that as the "input" for another function is important in contextual problems. Students can consider specific numerical examples and also more general examples when we use the expression of one function as the input for another. Using algebra to simplify the composition of the two functions can provide some practice in these algebraic skills. ● Students will also explore the connection between composition and the domain of a composite function. They can revisit the algebra and graphical methods for finding domain from Unit 1. ● Students use what they learn about composition of functions to further explore inverse

functions and use composition to show that one function is the inverse of another.
Important Considerations: ● In M3, students “Build new functions from existing functions. Find an inverse function. Understand the inverse relationship between exponential and logarithmic, quadratic and square root, and linear to linear functions and use this relationship to solve problems using tables, graphs, and equations. Determine if an inverse function exists by analyzing tables, graphs, and equations. If an inverse function exists for a linear, quadratic and/or exponential function, f, represent the inverse function, $f{-}1$, with a table, graph, or equation and use it to solve problems in terms of a context.” (NC.M3.F-BF.4) ● Composition requires some more in depth instruction that then can be revisited throughout each function-type cluster. ● The idea of composition (5.4) can be introduced with a contextual problem (i.e. Kohl’s cash problem- does the order matter?) Then, move into procedural standards (5.1, 5.2, 5.3). Standard 5.4 would be revisited in a more in-depth application problem after procedures are mastered. ● Students should explore ideas of composition and inverse both algebraically, tabular, and graphically.
Global Perspectives: In Calculus, it’s important for students to understand the composition and decomposition of functions when learning how to apply the Chain Rule for evaluating derivatives of compositions of functions. Inverse functions play an important role in considering the relationship between exponential and logarithmic functions and then later in the year when students study inverse trig functions. Understanding the graphical relationship between a function and its inverse and the idea of restricting the domain of a function so that is one-to-one will be important later in the course as students study trig functions.
Formal Assessments/Tasks:

Unit Name: Exponentials and Logarithms
Duration: **Block:** 3.5 - 4 weeks 18-20 days including assessment
Standards (Content): F.6.1: Use algebraic representations to build recursive functions. F.6.2: Construct a recursive function for a sequence represented numerically. F.4.2: Integrate information to build exponential functions to model phenomena involving growth or decay. F.4.1: Interpret algebraic and graphical representations to determine key features of exponential functions. *Key features include: domain, range, intercepts, intervals where the function is increasing, decreasing, positive or negative, concavity, end behavior, limits, and asymptotes.* F.4.7: Construct graphs of transformations of power, exponential, and logarithmic functions showing key features. F.4.3: Interpret algebraic and graphical representations to determine key features of logarithmic functions. *Key features include: domain, range, intercepts, intervals where the function is increasing, decreasing, positive or negative, concavity, end behavior, continuity, limits, and asymptotes.* A.2.1 Use properties of logarithms to rewrite expressions. A.2.2: Implement properties of exponentials and logarithms to solve equations. F.4.4: Implement graphical and algebraic methods to solve exponential and logarithmic equations in context with support from technology. F.5.1: Implement algebraic procedures to compose functions. F.5.6: Implement algebraic and graphical methods to find an inverse function of an existing function, restricting domains if necessary. F.5.7: Use composition to determine if one function is the inverse of another function.
Mathematical Practices: **1. Make sense of problems and persevere in solving them** 2. Reason abstractly and quantitatively **3. Construct viable arguments and critique the reasoning of others** **4. Model with mathematics** **5. Use appropriate tools strategically** 6. Attend to precision 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

What is the Mathematics?

- What students know about transformations of functions from M2-M3 should be applied here to exponential and logarithmic functions.
- The recursion topics include arithmetic and geometric growth and a combination of these exclusively.
- Students use properties of exponents and logs to rewrite expressions, solve equations, and apply them to contextual situations.
- Power functions are included after students study the graphs of exponential functions to compare rates of growth and understand the difference between characteristics of power functions and exponential functions, including equations, graphs, and tables.
- Students are asked to *both* build models *and* to analyze models given to them in context. Technology is used when appropriate.
- Throughout the unit, the key features listed in 4.1 and 4.3 should be addressed, but every key feature does not need to be included in every problem.
- Limits should be formalized to describe the end behavior of exponential and log functions and the behavior of logs close to the vertical asymptote. This builds on the previous emphasis on the qualitative behavior of functions. Limit notation will be part of this formalization.
- For the benefit of student learning and engagement, utilize multiple representations (tabular, algebraic, graphical) to complement each standard within this unit.

Important Considerations:

- The order for standards should begin with the use of recursion to build exponential functions, then graphing exponential functions, then building exponential models for contextual problems.
- Students study the behavior of power functions and compare that behavior to the behavior of exponential functions before moving to logarithms.
- Students solve exponential equations using technology (without the use of logarithms). Then, students should be introduced to logarithms using the fact that the logs are inverses of exponential functions.
- The properties of logs should be introduced later and then students can use logs and the properties of logs to solve exponential and log equations both in theoretical and contextual problems. Compositions can be used to verify that logs and exponential functions are inverses of one another.
- In Math 3, students "Build a function that models a relationship between 2 quantities" (M3.F-BF.1), including polynomial and exponential functions "with real solutions, given a graph, a description of a relationship, or ordered pairs (include reading these from a table)." (BF.1a). This unit builds on this foundation to further explore key

features of these functions, compare exponential functions to power functions, and use exponential functions to develop an understanding of logarithmic functions. ● While it is important for students to be familiar with algebraic procedures, the verb implement refers to using a procedure with an unfamiliar task.
Global Perspectives: With the understanding of exponential functions, students are able to graph curves that could describe/model real-world situations such as the population of bacteria or the temperature of an object over time. Using their knowledge of the end behavior of an exponential function, students can describe that end behavior of the model in context and use that information to find parameter values for the exponential model. These applications are often studied in Calculus where students are asked to both predict the long-term behavior of a function (limits) and the local behavior such as whether the function is increasing or decreasing or concave up or down.
Formal Assessments/Tasks: Medicine task Logistic Growth Model (Illustrative Mathematics) Feel the Noise (PBL unit on logarithms)

Unit Name: Trigonometry

Duration:
Block: 4-4.5 weeks
22-24 days including assessment

Standards (Content):
F.2.2: Explain how the symmetry of the unit circle is related to the periodicity of trigonometric functions.
F.2.1. Use a unit circle to find values of sine, cosine, and tangent for angles in terms of reference angles.
F.1.1 Interpret algebraic and graphical representations to determine key features of transformed sine and cosine functions. *Key features include: amplitude, domain, midline, phase shift, frequency, period, intervals where the function is increasing, decreasing, positive or negative, relative maximums and minimums.*
F.1.3: Integrate information to build trigonometric functions with specified amplitude, frequency, period, phase shift, or midline with or without context.
F.1.4: Implement graphical and algebraic methods to solve trigonometric equations and inequalities in context with support from technology.
F.1.2 Interpret algebraic and graphical representations to determine key features of tangent, cotangent, secant, and cosecant. *Key features include: domain, frequency, period, intervals where the function is increasing, decreasing, positive or negative, relative maximums and minimums, and asymptotes.*
F.3.3: Implement the Pythagorean identity to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
F.5.6: Implement algebraic and graphical methods to find an inverse function of an existing function, restricting domains if necessary.
F.3.1: Implement a strategy to solve equations using inverse trigonometric functions.
A.2.3 Implement properties of trigonometric functions to solve equations including inverse trigonometric functions, double angle formulas, and Pythagorean identities.
F.3.2: Implement Law of Sines and Law of Cosines to solve problems.

Mathematical Practices:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

What is the Mathematics?

- Student will have prior knowledge of special values for sine and cosine associated with the 30, 45, and 60-degree angles in the unit circle.
- Solving trig equations is important in that it helps students gain some experience identifying the algebraic structure of these equations. They then use their knowledge of the unit circle to solve equations and graph trig functions.
- After developing a deep understanding of the graphs of sine and cosine and their transformations, students will use these functions to build models in context.
- Later, with the use of inverse trigonometric functions, students will use technology to solve these types of equations when the exact solutions are not possible to find analytically. Students moving on to Calculus will also benefit from analyzing the graph of the inverse tangent function as an extension.
- The graph of tangent, cotangent, cosecant and secant are studied later in the unit.
- Limits will be revisited to describe the behavior of tangent, cotangent, cosecant and secant close to the vertical asymptotes. This builds on the previous emphasis on the qualitative behavior of functions. Limit notation will be part of this formalization.

Important Considerations:

- Law of cosines goes last
- In Math 3, students learn about sine and cosine functions in the standard M2.F-TF.2: Extend the domain of trigonometric functions using the unit circle. In particular, this standard states, "Build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions. Interpret the sine function as the relationship between the radian measure of an angle (or arc length) formed by the horizontal axis and a terminal ray on the unit circle and its *y* coordinate. Interpret the cosine function as the relationship between the radian measure of an angle (or arc length) formed by the horizontal axis and a terminal ray on the unit circle and its *x* coordinate." Therefore, we recommend starting with the unit circle.
- Use unit circle to build graphs
- Periodicity of the cosine and sine and symmetry of the unit circle go together

Global Perspectives:

In this unit, students explore the idea that the graphs of sine, cosine and tangent can be built from our knowledge of the sine and cosine of the special angles measured in radians. In Calculus, students will be asked to differentiate and integrate transformations of the trig functions. They will also be asked to find extreme values and points of inflection for transformations of the trig functions. In Calculus, students will study the rates of change of the trig functions and find the extrema of these functions. They may use trig identities to aid in the analytic evaluation of integrals of trig functions.

Formal Assessments/Tasks:
As the Wheel Turns (Illustrative Mathematics)
Intro to periodic functions: Model Airplane Acrobatics (Illustrative Mathematics)
Your Hood (PBL on Law of Sines and Law of Cosines

Noise Cancelling Headphones (from McCulloch, A. W., Whitehead, A., Lovett, J. N., & Patterson, B. (2017). Tuning out the world with noise cancelling headphones. *The Mathematics Teacher, 110(8)*, 606, 611)

Unit Name: Parametric Equations

Duration: **Block:** 0.6 week 3 days
Standards (Content): F.7.1: Implement algebraic methods to write parametric equations in context. F.7.2: Implement technology to solve contextual problems involving parametric equations. A.2.4 Implement algebraic techniques to rewrite parametric equations in cartesian form by eliminating the parameter.
Mathematical Practices: 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
What is the Mathematics? ● Students will have prior knowledge of creating linear equations from a description of a context. ● Students will use their knowledge of linear and quadratic functions to create models for the horizontal and vertical positions of an object. Thus creating parametric equations to represent the object's position. ● Students will use technology to create graphs for the position of an object when given the parametric equations that represent an object's position. ● Students will use their algebraic skills to eliminate the parameter and rewrite a set of parametric equations as y = f(x). ● By using technology to create the graphs of the positions of objects given as parametric equations, students can answer questions related to the position of these objects.
Important Considerations: ● Because using parametric equations can produce graphs of curves that are not functions, students can explore a variety of graphs using parametric equations (and are not limited to the more typical graphs. ● In using technology to create the graphs of parametric equations, students can consider that a graph can have direction and that the parameter, in contextual problems, represents time.

- The graphs of the parametric curves can be connected to previous knowledge such as the graphs of conic sections such as circles and ellipses.

Global Perspectives:
This introduction to parametric equations can serve as motivation for students to explore concepts in Physics and later in calculus. In calculus, students will see how we can explore the rates of change of the horizontal and vertical functions individually and how those rates of change affect the rate of change of the graph of y = f(x). For example the t values for which dx/dt = 0 can lead us to the places on the graph where the tangent line to the graph is vertical.

Formal Assessments/Tasks:
One Shot (PBL unit on parametric equations and vectors)

Unit Name: ACT Prep

Duration: **Block**: 0.4 week 2 days
Standards (Content): N.1.1: Execute the sum and difference algorithms to combine complex numbers. N.1.2: Execute the multiplication algorithm with complex numbers. N.2.1: Execute the sum and difference algorithms to combine matrices of appropriate dimensions. N.2.2: Execute associative and distributive properties to matrices. N.2.3: Execute commutative property to add matrices. N.2.4: Execute properties of matrices to multiply a matrix by a scalar. N.2.5: Execute the multiplication algorithm with matrices. N.3.1: Represent a vector indicating magnitude and direction. N.3.2: Execute sum and difference algorithms to combine vectors.
Mathematical Practices: 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
What is the Mathematics? ● In Math 2, students learned about complex numbers in the context of simplifying expressions with a negative value under a square root and solving quadratic equations. In this course, students will simplify complex numbers using addition, subtraction, and multiplication. ● Students will use the operations of addition, subtraction, multiplication and scalar multiplication to combine matrices.
Important Considerations: ● This unit is purely for the *procedural knowledge* of complex numbers, matrices, and vectors. ● This topic may be best placed in the Extending Polynomial & Rational Functions, with the study of functions that contain imaginary solutions. ● The connection between parametric equations and vectors may be made in the

Parametric Equation Cluster. • These standards are included across fourth level courses in order to prepare students for the ACT due to the state legislature. • ***NOTE:*** *This unit does not have to be taught at the end of the course. These standards can be scattered anywhere it works for you to teach them. They do not fit necessarily in any of the other units cohesively, but that does not mean one could not include them as time permits.*
Global Perspectives:
Formative Assessments/Tasks: Neat Matrix Multiplication Task: https://www.openmiddle.com/matrix-multiplication/ Complex Numbers: https://www.openmiddle.com/complex-number-products/ Possible Homework Assignment: Add, Subtract, Multiply Complex Numbers Matrix Multiplication Task Keeping Secrets (PBL unit on matrices) One Shot (PBL unit on parametric equations and vectors) Complex Number Patterns (Illustrative Mathematics) Vertex of a Parabola with Complex Roots (Illustrative Mathematics)