NC Math 3 – Polynomial Functions

ALGEBRA, FUNCTION, AND NUMBER

The third unit of the NC Collaborative Pacing Guide extends students' prior conceptions of function and algebra to polynomials of higher degree by focusing on:

- building polynomial functions in context, often based on geometric figures (F-BF.1, G-GMD.3, G-MG.1, F.BF.1);
- key features and representations by comparing and transforming functions (F.IF.4,.7,.9; F.BF.3);
- connecting the concepts of roots of a polynomial and zeros of a function by applying the Fundamental Theorem of Algebra (*N-CN.9*); and
- combining algebraic methods of solving equations with broad generalizations about the existence of solutions by applying relevant theorems and algorithms (*A-APR.2,.3*; *N-CN.9*).

BUILDING FROM NC MATH 2

In NC Math 2, students engage in an in-depth look at quadratic functions with real coefficients. They connect algebraic solutions to graphical intercepts, and generalize the shapes of quadratics' graphs based on the structure in their different forms (standard, factored, vertex).

Additionally, students build quadratic formulas from context, generalizing patterns that involve a product of two binomials (hence the resulting x^2 term). In NC Math 3, this work is extended to higher degree polynomials, therefore, teachers could begin this unit with a three-dimensional growing pattern or model geometric figures that generalize to a cubic polynomial (e.g. Painted Cube).

In NC Math 2, students develop an understanding of complex numbers as a way to express solutions to quadratic equations that do not produce x-intercepts for the function. It is important in this unit that teachers continue to build students' understanding of ***real numbers*** - complex numbers that do not have an imaginary factor (e.g. rational numbers, π, $\sqrt{2}$) and their superset, ***complex numbers*** – numbers with real and/or imaginary factors (e.g. i, $\frac{7}{5}$, 1, $5+2i$) to support students in undestanding and applying the Fundamental Theorem of Algebra to determine the number and types of solutions for polynomial functions.

THE FUNDAMENTAL THEOREM OF ALGEBRA

Wait, wait, wait,… a theorem that is *fundamental* to *Algebra*, but we are just now finding out about it? The label of "fundamental", in this case, is not to be synonymous with "basic".

The theorem provides the assurance that the number of complex roots of a non-linear polynomial will equal the degree of the polynomial, if you include multiplicity when counting roots. The theorem is also an example of what's known as an *existence proof*. That is, it guarantees the roots exist, without telling you what they are or how to find them.

As with *The Fundamental Theorem of Arithmetic* (every integer greater than 1 has a unique prime factorization) and *The Fundamental Theorem of Calculus* (if a continuous function has an antiderivative, the antiderivative can be used to calculate integral values) students can use the result of *The Fundamental Theorem of Algebra*, without knowing a proof of the theorem.

DISCUSS WITH YOUR COLLEAGUES

How do **you** know this theorem is true?
What convinces **you**?
If you are still a skeptic, here is a link to a Numberphile Video proof as a place to start.

INVESTIGATING POLYNOMIAL FUNCTIONS

NC Math 1, 2, and 3 all contain *F-IF.*7 and *F-IF.*9 standards, which appear in five of the eight units in NC Math 3. These standards require students to analyze the characteristics of domain, range, intervals of increase and decrease, extreme values, end behavior, and compare functions by their key features as found in different representations.

Focusing on graphical representations, students can generalize the possible shapes of the graphs of polynomial functions based solely on the degree of the polynomial and the sign value of the leading coefficient. Knowing the roots of the polynomial and their multiplicities will help students better locate their sketch of the graph of the polynomial function within the coordinate plane.

Using a graphing utility is a great way to efficiently create multiple graphs so that students can look for patterns in order to generalize shape. The online calculator Desmos hosts a Teacher Desmos site offering lessons that are classroom ready (visit the Teacher Desmos site and search for *polynomials*).

Research has shown that when examining graphical representations, students may have difficulty connecting graphs to the context they represent (Piez & Voxman, 1997). Additionally, students may overgeneralize characteristics of parameters across function families when engaging with symbolic representations and have difficulty connecting key features of functions recognized in tables to other representations (Wilson, 1994). Therefore, in instruction, it is important to attend to multiple representations to support students in developing strong connections across representations and their relationships to the context of the problem.

CONNECTING TO STUDENTS' PRIOR UNDERSTANDING

The development of standard algorithms for arithmetic are used based on their efficiency and it is a goal that by the end of 6th grade, North Carolina students can fluently divide using the *standard algorithm* for division. Research has shown that when students engage in arithmetic using only standard algorithms they may lack the underlying concepts of base-10 numbers and relationships between multiplication and division (Ambrose et al., 2003).

DISCUSS WITH YOUR COLLEAGUES

- *How could you use equivalencies to* $\frac{9}{8}$, $\frac{7}{8}$, *and* $\frac{9}{9}$ *to make sense of* $\frac{x+1}{x}$, $\frac{x-1}{x}$, *and* $\frac{x^2+1}{x^2+1}$?
- *Before we divide, which of these quotients should be greater than 1? How do you know?*

$$\frac{x+2}{x-1} \quad \frac{x^2-1}{x^2} \quad \frac{x^2+4x+4}{(x+2)^2} \quad \frac{x^3-1}{x-1}$$

It's important to remember that students may enter our classroom in NC Math 3 with unrefined conceptions of division that may affect the ways in which they engage with polynomial division or synthetic division (A-APR.2,.3). Teachers can support students in building toward an understanding of what polynomial division is by carefully sequencing well-crafted simple examples that connect to thinking about fractions.

Teachers can also craft examples that build upon students' understanding of the inverse relationship between the operations of multiplication and division.

DISCUSS WITH YOUR COLLEAGUES

- *Knowing that* $x^2 - x - 6 = (x-3)(x+2)$, *what is the quotient of* $\frac{x^2-x-6}{x+2}$?
- *Since* $\frac{2x^2-2}{x-1} = 2x+2$, *then the product* $(x-1)(2x+2)$ *would be equal to* _________.
- *What instructional decisions can you make in this unit to ensure students develop both a procedural and conceptual understanding of polynomials?*
- *How do the examples provided in this brief support you in understanding ways you can support your students?*

References

Ambrose, R., Baek, J.-M. & Carpenter, T.P. (2003). Children's invention of multiplication and division algorithms. In A. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Recent research and theory*. Mahwah, NJ: Erlbaum.

Piez, C. M., & Voxman, M. H. (1997). Multiple representations—Using different perspectives to form a clearer picture. *The Mathematics Teacher*, *90*(2), 164-166.

Wilson, M. R. (1994). One preservice secondary teacher's understanding of function: The impact of a course integrating mathematical content and pedagogy. *Journal for Research in Mathematics Education*, 346-370.

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SUGGESTED CITATION

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